CS202A: Introduction to Mathematical Logic. Assignment 1

27 October 2016

Due on: 4 November 2016. Please hand over your assignment to the TA in the class.

1. There is an alternative approach to treating negation: one simply bans the operator $\neg$ and thinks of $\phi \to \bot$ as $\neg\phi$. Propose a natural deduction system with this approach to negation that is both sound and complete. Outline a soundness and completeness proof of your proposed system. (If in your proofs you would like to include some parts of the proofs that we learnt in class, you need to just refer to those parts, there is no need to re-do those proof segments.)

2. Suppose to our propositional logic, we add one more connective: $\leftrightarrow$ with the intention $\phi \leftrightarrow \psi$ to abbreviate $\phi \to \psi \ \wedge \ \psi \to \phi$. Give appropriate introduction and elimination rules for $\leftrightarrow$ and show that the new natural deduction system is also sound, by appropriately modifying the soundness proof that we have learnt.

3. Let 2CNF denote the subset of the CNF formulas where each clause has *exactly* two literals. Provide a satisfiability decision algorithm for 2CNF. (It is possible to check 2CNF satisfiability efficiently, that is, in polynomial time. You should try to provide such a decision algorithm.) Prove correctness of your decision algorithm.