Data Structures and Algorithms

(CS210A)

Semester I – **2014-15**

Lecture 11:

- **Queue : a new data Structure :**
- **Finding shortest route in a grid in presence of obstacles**

Queue: a new data structure

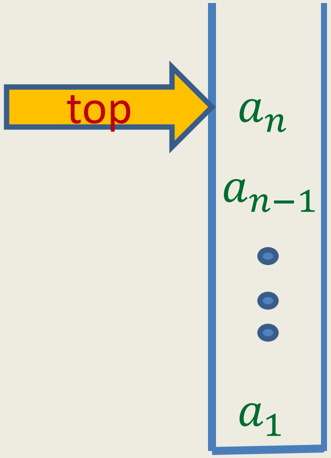
Data Structure Queue:

- Mathematical Modeling of Queue

- Implementation of Queue using arrays

Stack

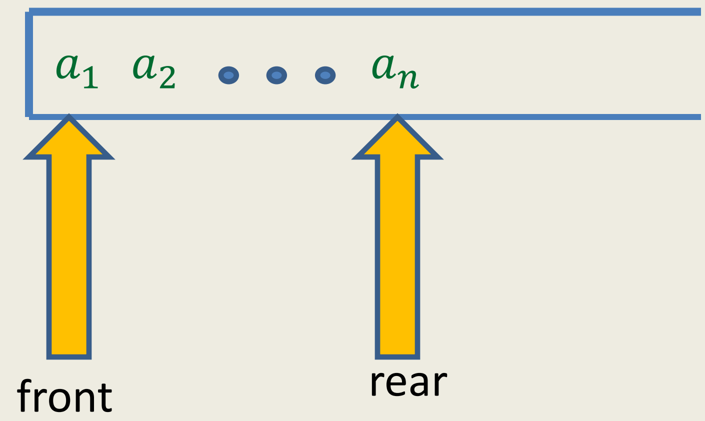
A special kind of list where all operations (insertion, deletion, query) take place at one end only, called the **top**.



Behavior of Stack: **Last in First out (LIFO)**

Queue: a new data structure

A special kind of list based on "**First in First Out (FIFO)**" strategy.



Operations on a Queue

Query Operations

- **IsEmpty(Q)**: determine if **Q** is an empty queue.
- **Front(Q)**: returns the element at the front position of the queue.

  Example: If **Q** is $a_1, a_2, \ldots, a_n$, **then Front(Q)** returns a_1 .

Update Operations

- **CreateEmptyQueue(Q)**: Create an empty queue.
- **Enqueue(x,Q)**: insert **x** at the **end** of the queue **Q**.

  Example: If **Q** is $a_1, a_2, \ldots, a_n$, then after **Enqueue(x,Q)**, queue **Q** becomes

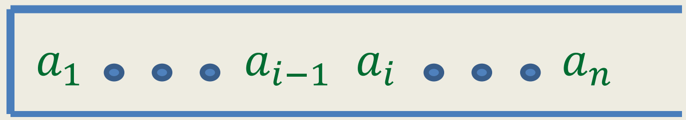
  $a_1, a_2, \ldots, a_n$, **x**

- **Dequeue(Q)**: return element from the **front** of the queue **Q** and delete it.

  Example: If **Q** is $a_1, a_2, \ldots, a_n$, then after **Dequeue(Q)**, queue **Q** becomes

  $a_2, \ldots, a_n$

How to access ith element from the front ?



- To access ith element, we **must** perform **dequeue** (hence delete) the first i-1 elements from the queue.

An Important point you must remember for every data structure

You can define any **new** operation only in terms of the primitive operations of the data structures defined during its modeling.

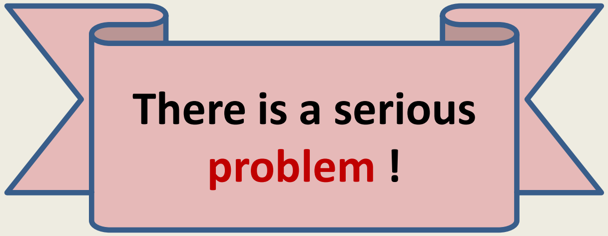
Implementation of Queue using array

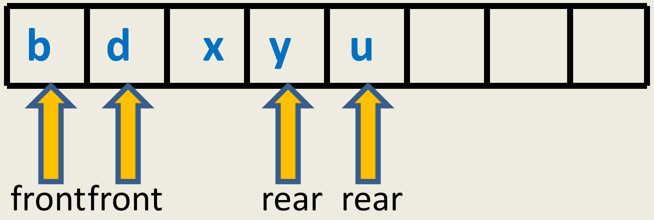
Assumption: At any moment of time, the number of elements in queue is n.

Keep an array of **Q** size n, and two variables front and rear.

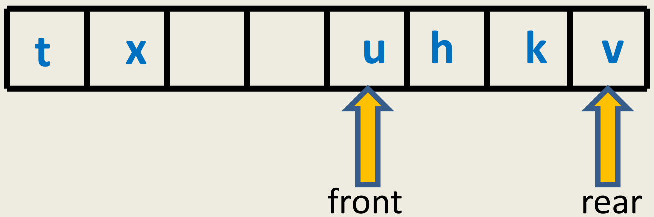
- front: the position of the **first** element of the queue in the array.
- rear: the position of the **last** element of the queue in the array.

```
Enqueue(x,Q)
{      rear ← rear+1;
   Q[rear]←x
}
Dequeue(Q)
{         x← Q[front];
    front← front+1;
    return x;}
```





Implementation of Queue using array



How to perform Enqueue(x,Q) ?

Implementation of Queue using array

Enqueue(x,Q)

```
{    rear ← rear+1 mod n ;
  Q[rear]←x
}
```

Dequeue(Q)

```
{       x← Q[front];
   front← (front+1) mod n ;
   return x;
}
```

IsEmpty(Q)

```
{   Do it as an exercise   }
```

Shortest route in a grid with obstacles

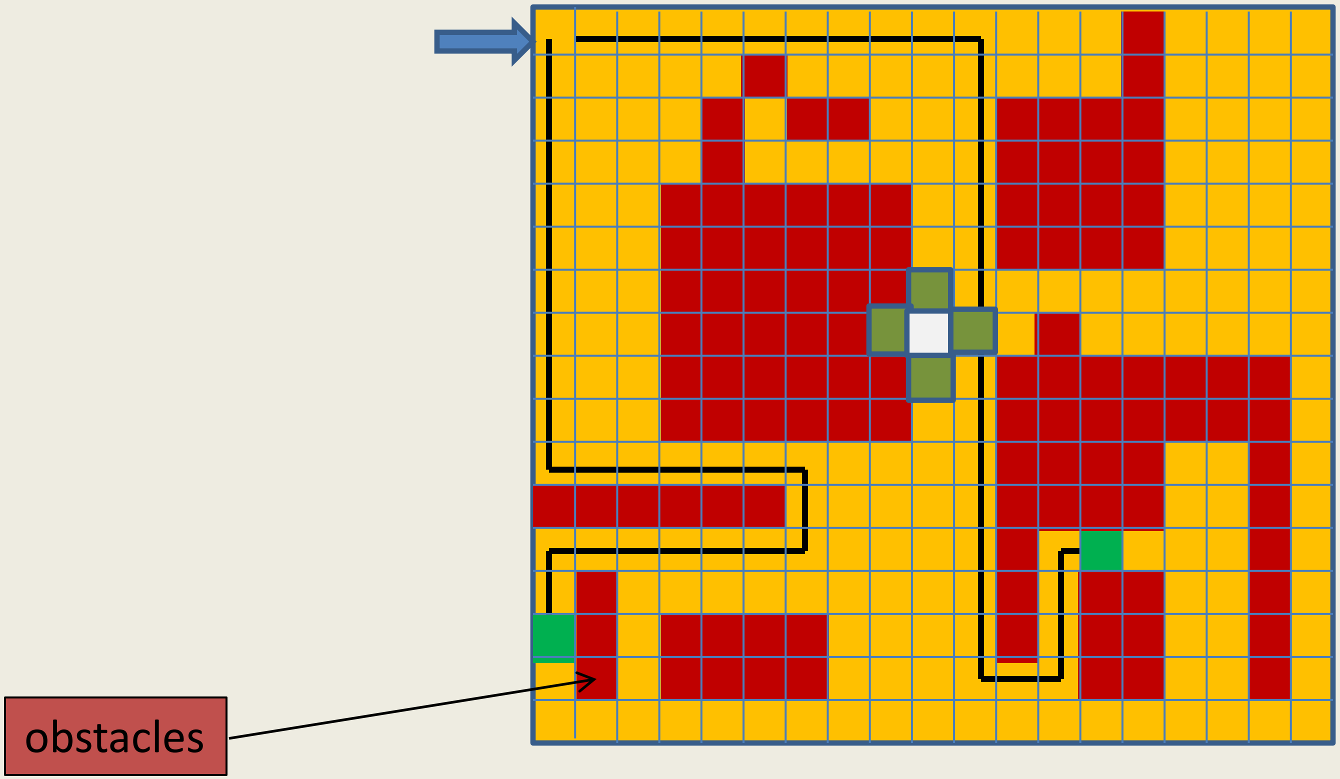
Shortest route in a grid

From a cell in the grid, we can move to any of its <u>neighboring</u> cell in one <u>step</u>.

Problem: From <u>top left corner</u>, find shortest route to each cell <u>avoiding</u> obstacles.

Input : a Boolean matrix G representing the grid such that

$G[i, j]$ = 0 if (i, j) is an obstacle, and 1 otherwise.

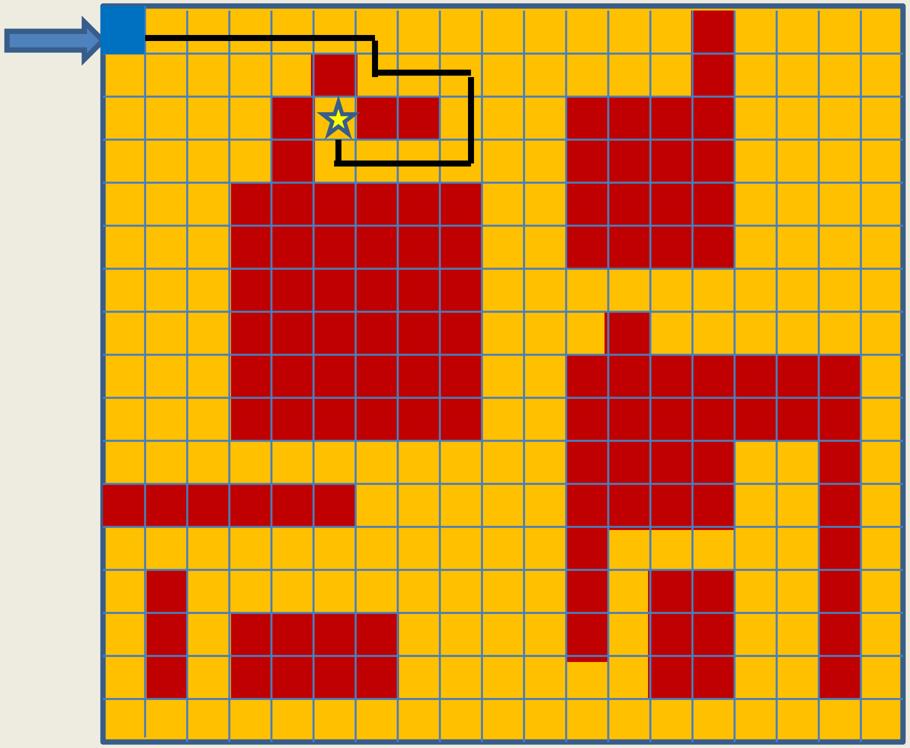


Step 1:

Realizing the nontriviality of the problem

Shortest route in a grid

nontriviality of the problem

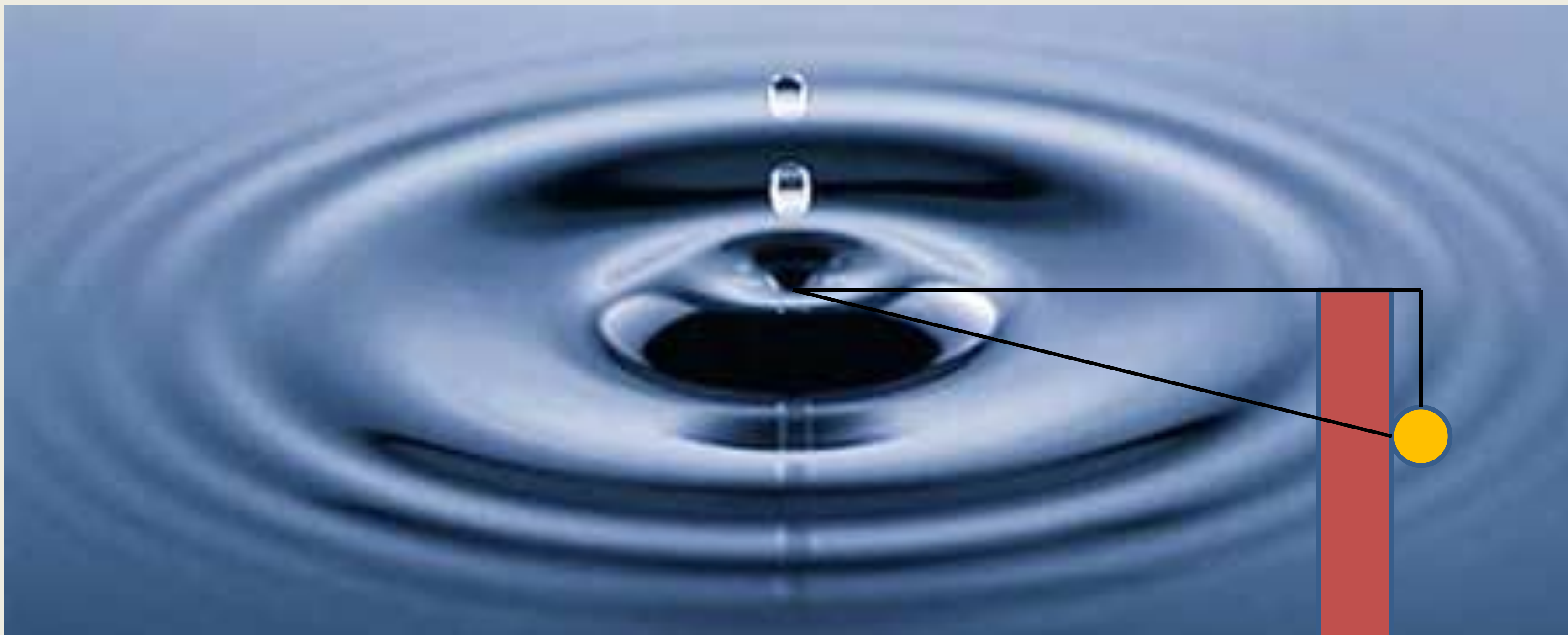


Definition: Distance of a cell **c** from another cell **c'** is the length (number of steps) of the shortest route between **c** and **c'**.

We shall design algorithm for computing distance of each cell from the start-cell.

As an exercise, you should extend it to a data structure for retrieving shortest route.

Get inspiration from nature

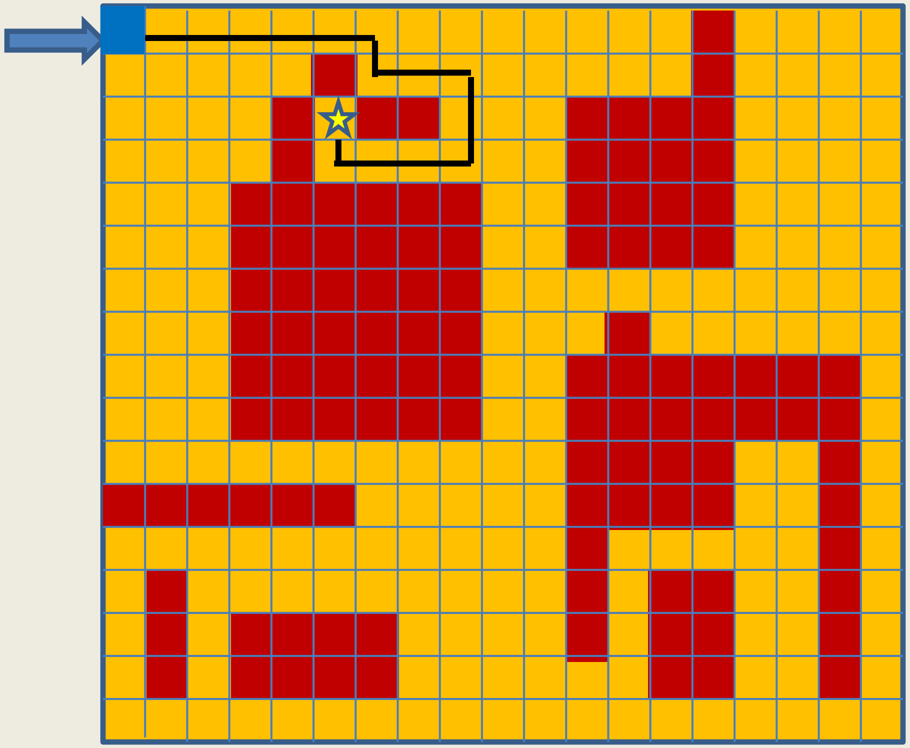


The ripples travels along the shortest route ?

Shortest route in a grid

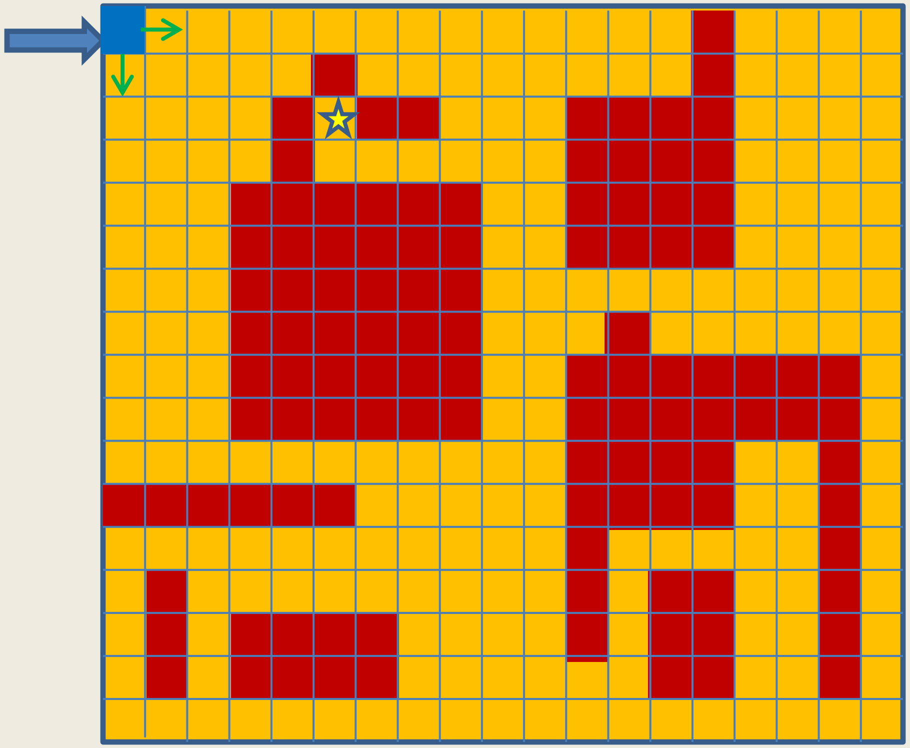
nontriviality of the problem

How to find the shortest route to ☆ in the grid ?

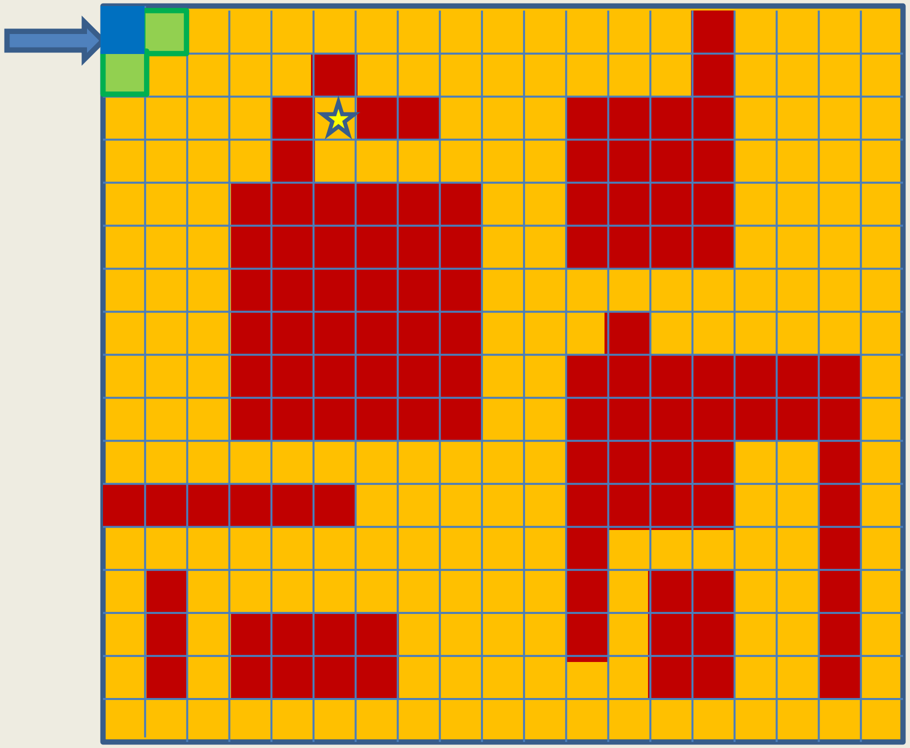


Create a ripple at the start cell and trace the path it takes to ☆

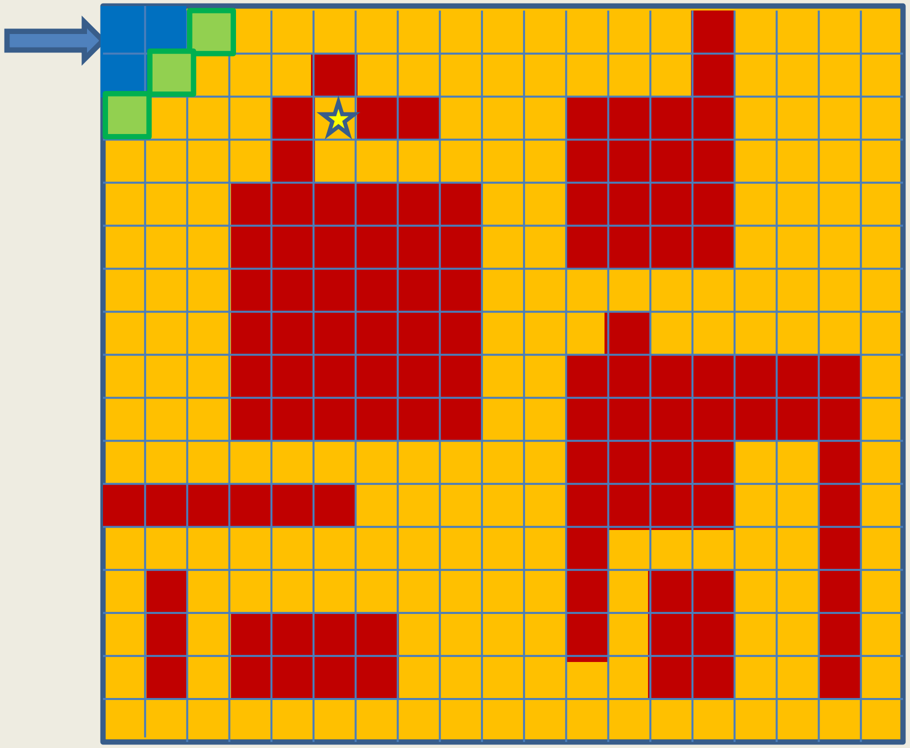
propagation of a ripple from the start cell



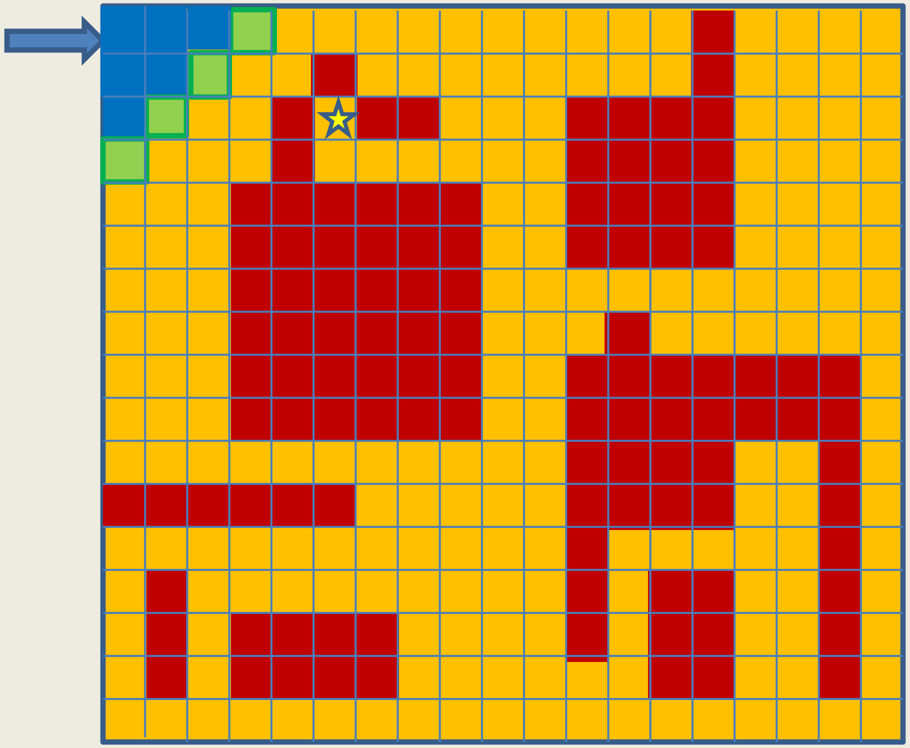
ripple reaches cells at distance 1 in step 1



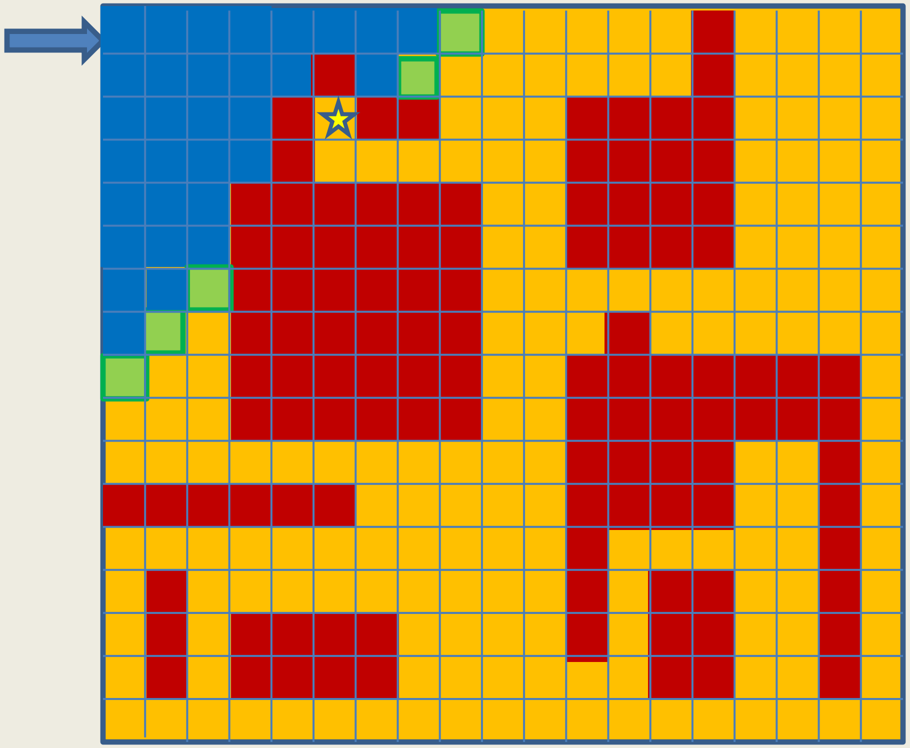
ripple reaches cells at distance 2 in step 2



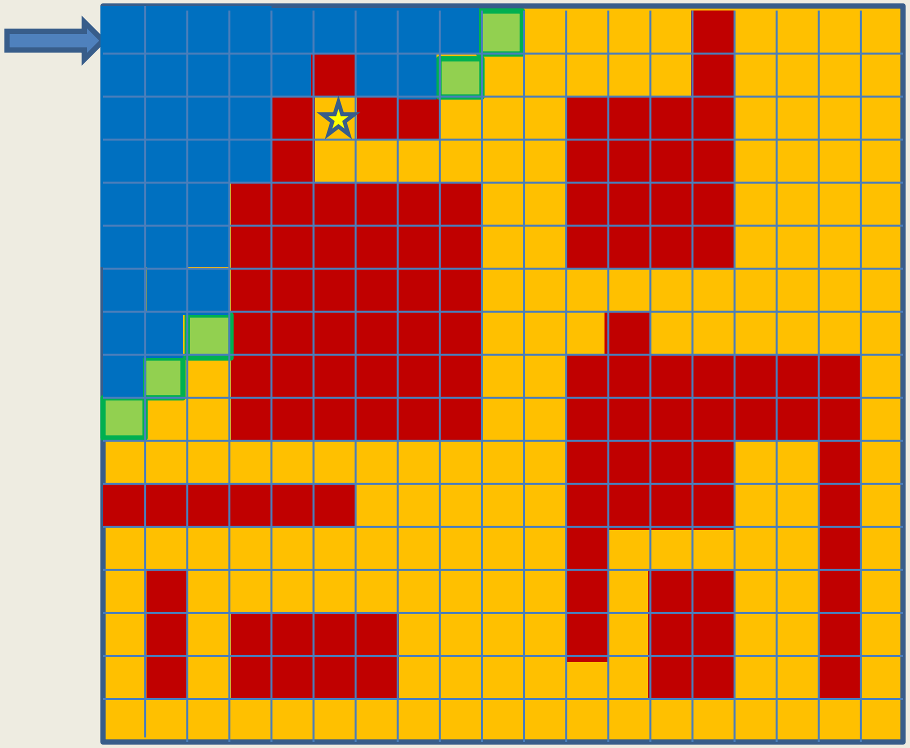
ripple reaches cells at distance 3 in step 3



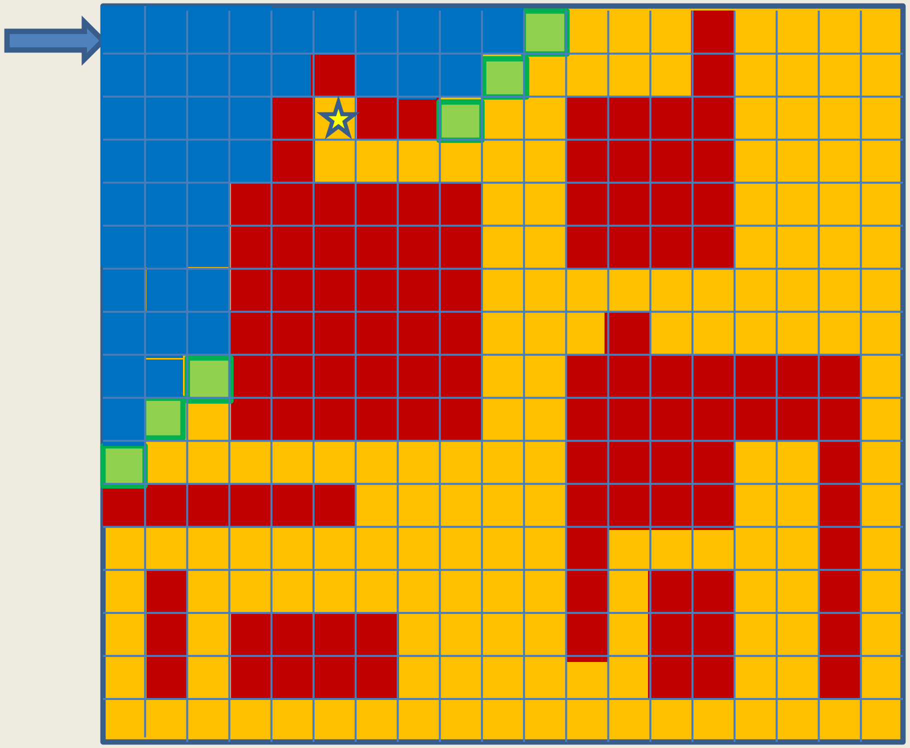
ripple reaches cells at distance 8 in step 8



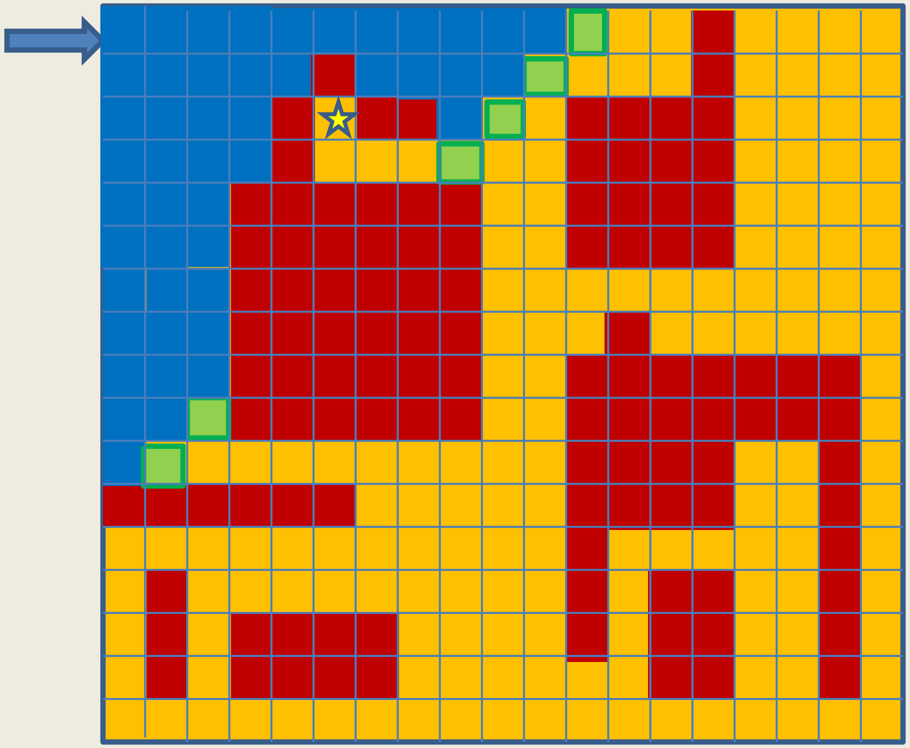
ripple reaches cells at distance 9 in step 9



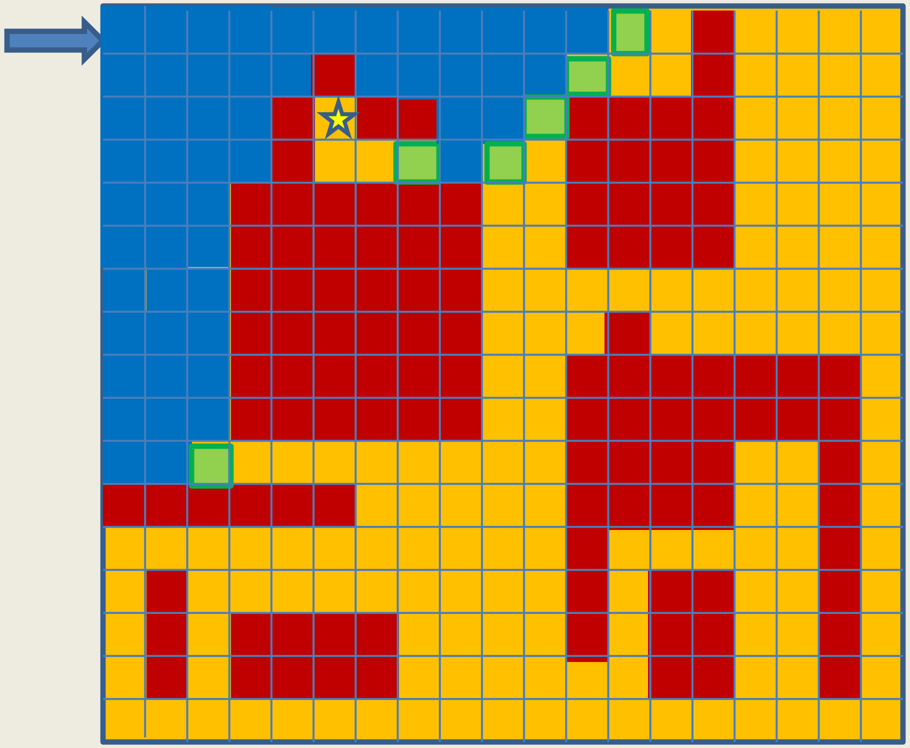
ripple reaches cells at distance 10 in step 10



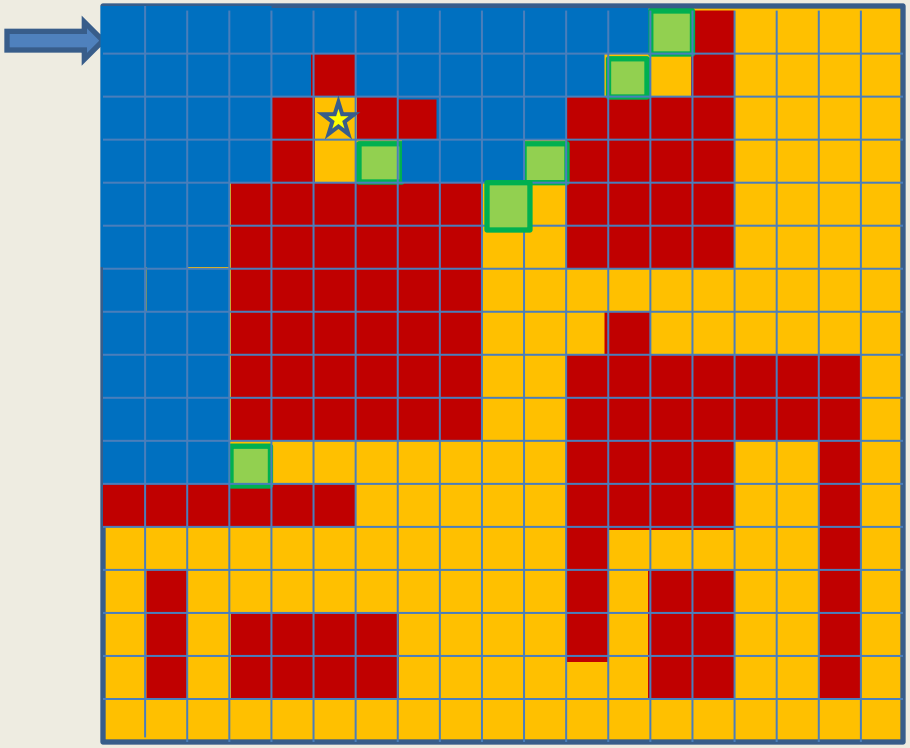
ripple reaches cells at distance 11 in step 11



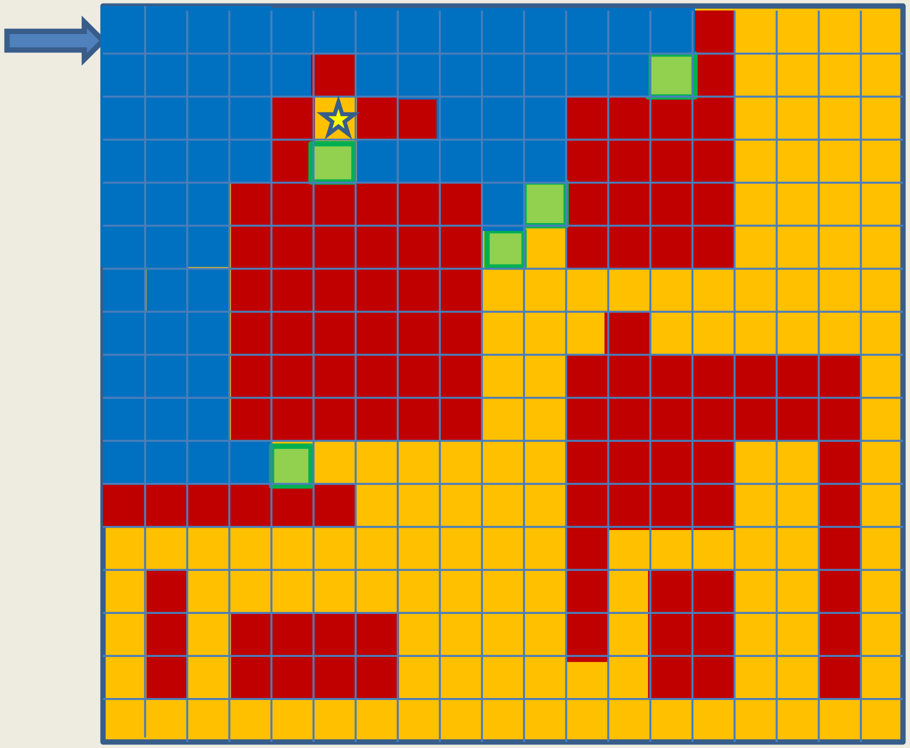
ripple reaches cells at distance 12 in step 12



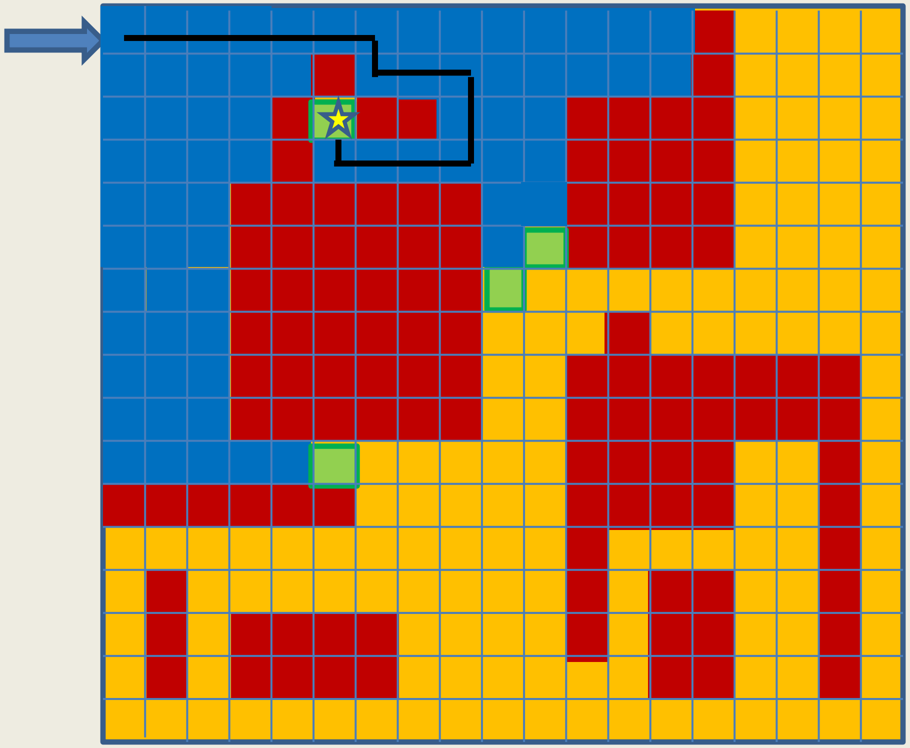
ripple reaches cells at distance 13 in step 13



ripple reaches cells at distance 14 in step 14



ripple reaches cells at distance 15 in step 15



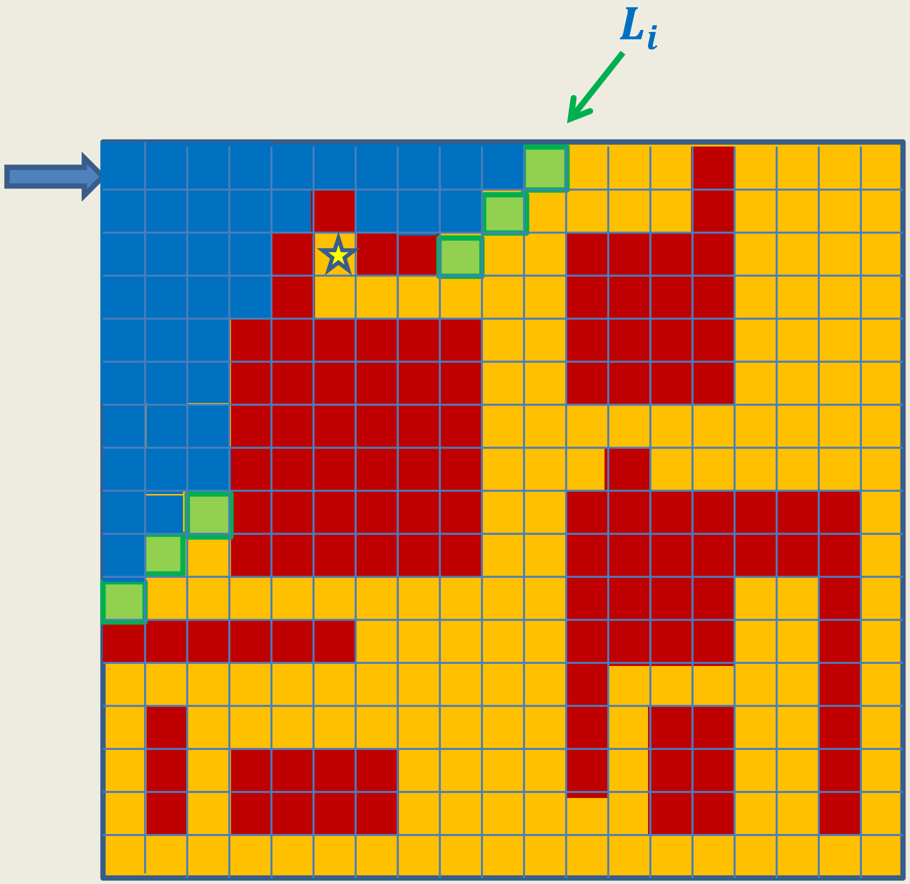
The route taken by ripple is indeed the shortest one.

Step 2:

Designing algorithm for distances in grid

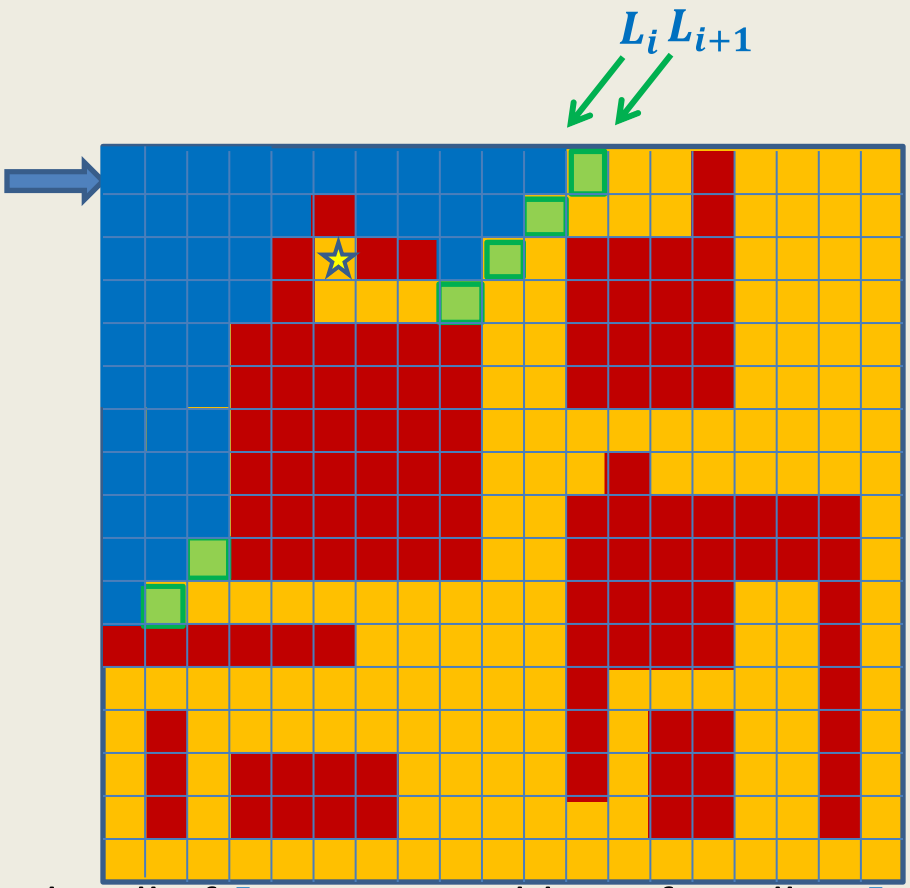
(using an insight into propagation of ripple)

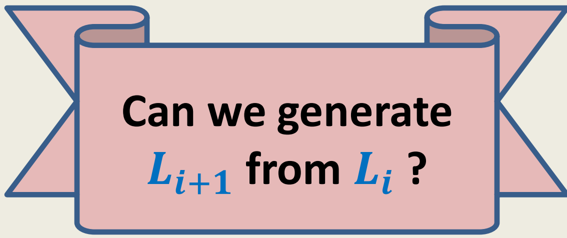
A snapshot of ripple after i steps



L_i : the cells of the grid at distance i from the starting cell.

A snapshot of the ripple after $i+1$ steps





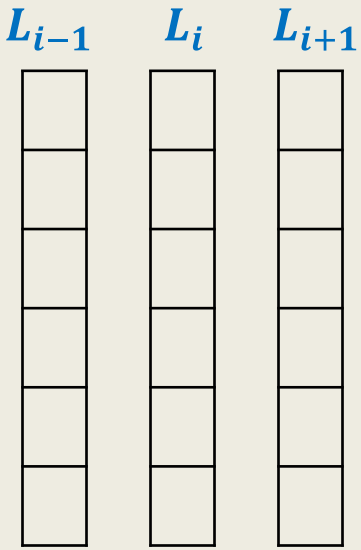
Observation: Each cell of L_{i+1} is a neighbor of a cell in L_i.

But every neighbor of L_i may be a cell of L_{i-1} or L_i or L_{i+1}.

How can we **generate** L_{i+1} from L_i ?

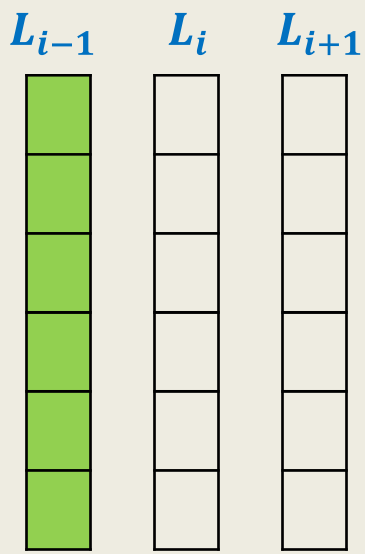
There is an **order** in which the cells are visited.

How can we generate L_{i+1} from L_i ?



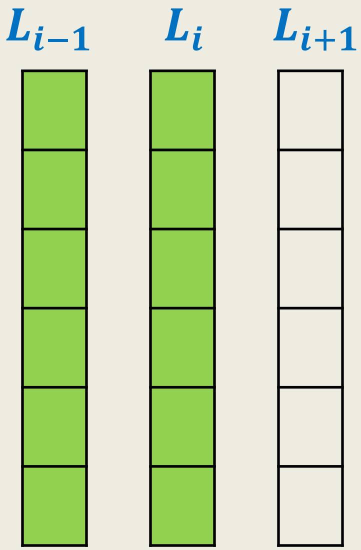
How can we generate L_{i+1} from L_i ?

First all cells of L_{i-1} get visited,



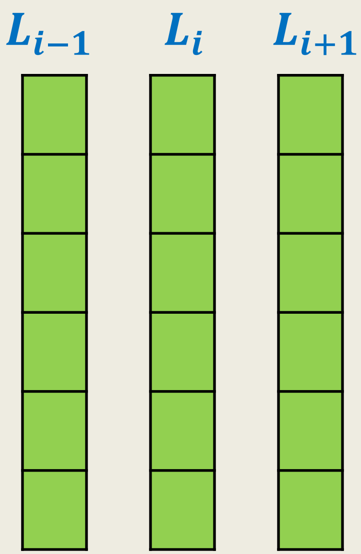
How can we **generate** L_{i+1} from L_i ?

First all cells of L_{i-1} get visited,
Then all cells of L_i are visited, and



How can we generate L_{i+1} from L_i ?

First all cells of L_{i-1} get visited,
Then all cells of L_i are visited, and
then all cells of L_{i+1} are visited.



So by the time all cells of L_i are visited, if a cell neighboring to a cell of L_i is unvisited, it must be a cell of L_{i+1}. ☺

Algorithm to compute L_{i+1} if we know L_i

```
Compute-next-layer(G, L_i)
{
   CreateEmptyList(L_{i+1});
   For each cell c in L_i
         For each neighbor b of c which is not an obstacle
         {      if (Distance[b] = ∞)
                {      Insert(b, L_{i+1});
                       Distance[b] ← i+1 ;
                }
         }
   return L_{i+1};
}
```

The first (not so elegant) algorithm

(to compute distance to all cells in the grid)

Distance-to-all-cells(**G**, $\mathbf{c}_0$)

{ $\boldsymbol{L_0}$ ← {$\mathbf{c}_0$};

For(***i*** = 0 to ??)

$\boldsymbol{L_{i+1}}$ ← **Compute-next-layer**(**G**, $\boldsymbol{L_i}$);

}

It can be as high as O(n^2)

The algorithm is not elegant because of

- So many temporary lists that get created.

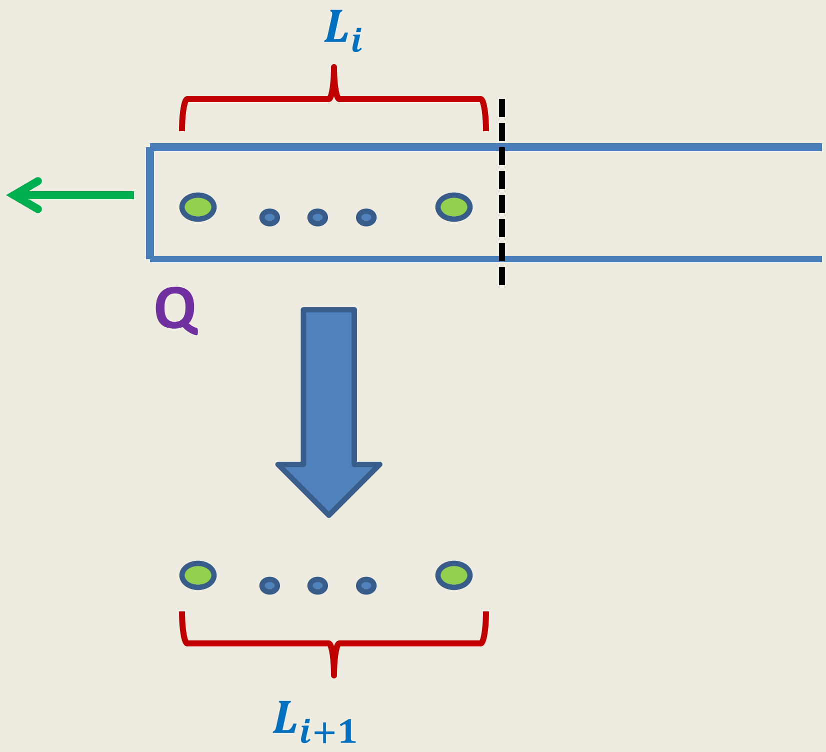
How to transform the algorithm to an elegant algorithm ?

Key points we have observed:

- We can compute cells at distance i+1 if we know all cells up to distance i.
- Therefore, we need a mechanism to enumerate the cells of the grid in **non-decreasing** order of **distances** from the start cell.

How to design such a mechanism ?

Keep a queue Q



An elegant algorithm

(to compute distance to all cells in the grid)

```
Distance-to-all-cells(G, c0)
    CreateEmptyQueue(Q);
    Distance(c0) ← 0;
    Enqueue(c0,Q);
    While( Not IsEmptyQueue(Q) )
    {         c ←Dequeue(Q);
              For each neighbor b of c which is not an obstacle
              {       if (Distance(b) = ∞)
                          {  Distance(b) ← Distance(c) +1 ;
                             Enqueue(b, Q); ;
                          }
              }
    }
```

Proof of correctness of algorithm

Question: What is to be proved ?

Answer: At the end of the algorithm,

Distance[c]= the distance of cell c from the starting cell in the grid.

Question: How to prove ?

Answer: By the principle of mathematical induction on

the distance from the starting cell.

Inductive assertion:

P(*i*): The algorithm correctly computes distance to all cells at distance ***i*** from the starting cell.

As an exercise, try to prove **P(*i*)** by induction on ***i***.