Data Structures and Algorithms

(CS210A)

Semester I – **2014-15**

Lecture 19:

Red Black tree (Final lecture)

- **9 types of operations, each executed in $O(\log n)$ time !**

Red Black tree

(Height Balanced BST)

Operations you already know

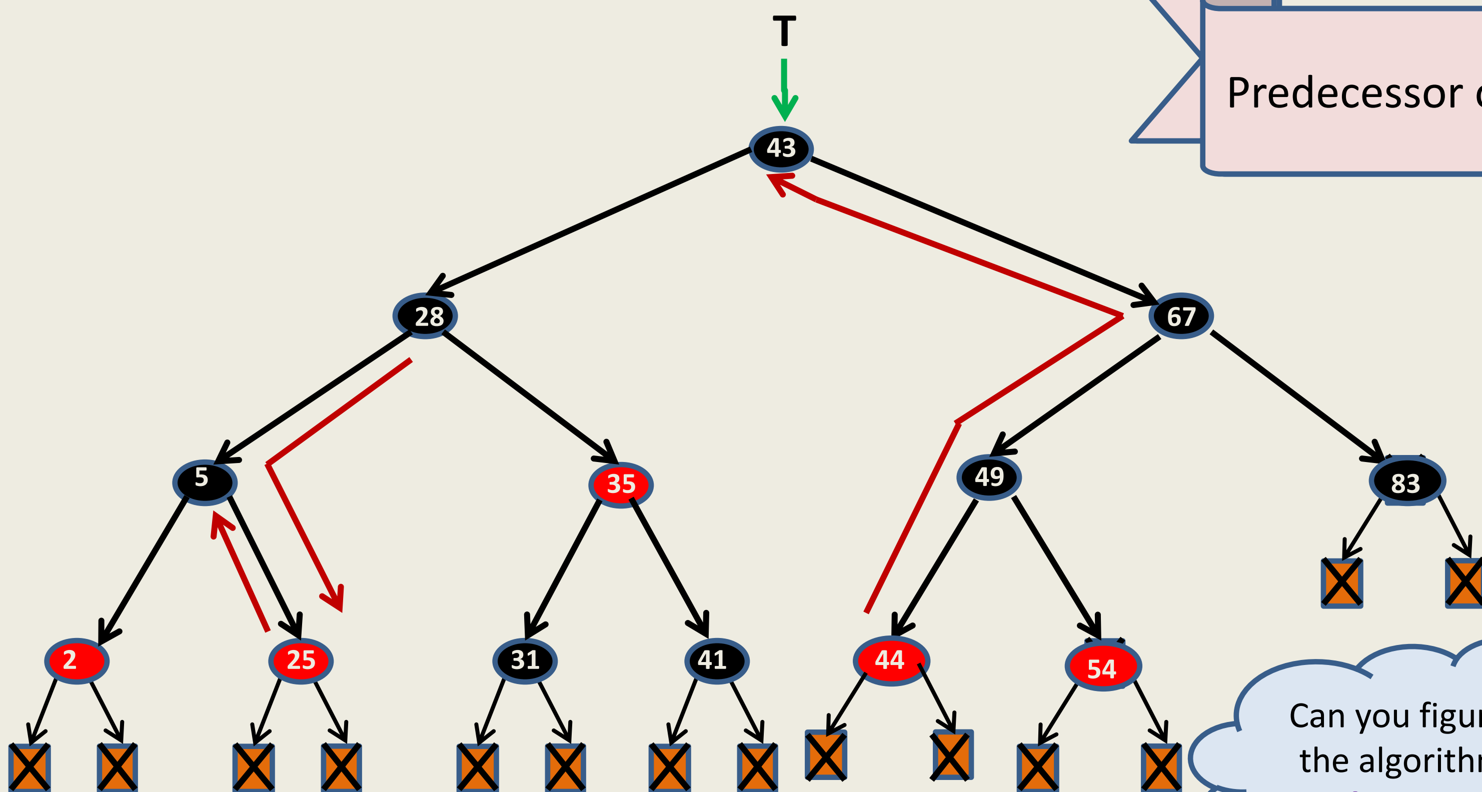
1. Search(T,x)
2. Insert(T,x)
3. Delete(T,x)
4. Min(T)
5. Max(T)

Every operation in O(log n) time.

Predecessor(T,x)

The **largest** element in **T** which is **smaller than x**

Predecessor(T,x)

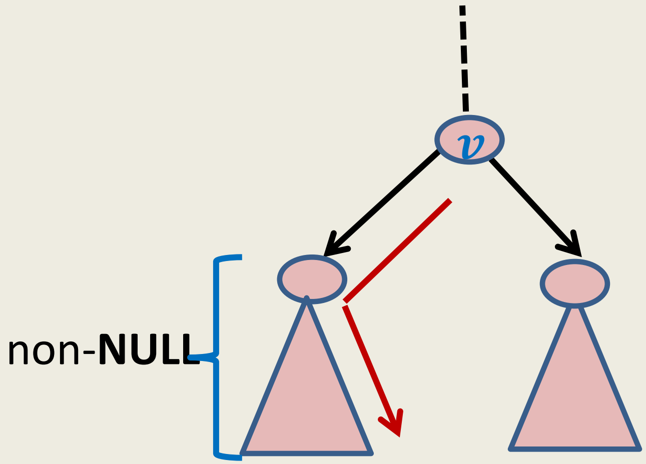


Predecessor of **25 ?**

Can you figure out the algorithm for Predecessor(T,x) ?

Predecessor(T,x)

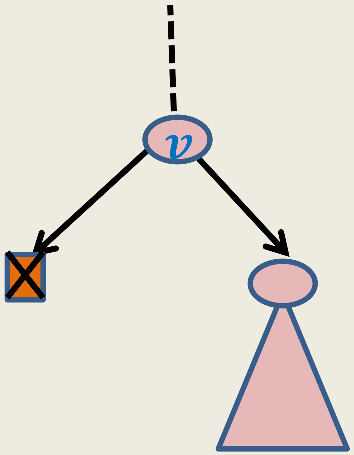
Let v be the **node** of **T** storing value x.



Case 1: **left**(v) <> **NULL** , then **Predecessor(T,x)** is **Max(left(v))**

Predecessor(T,x)

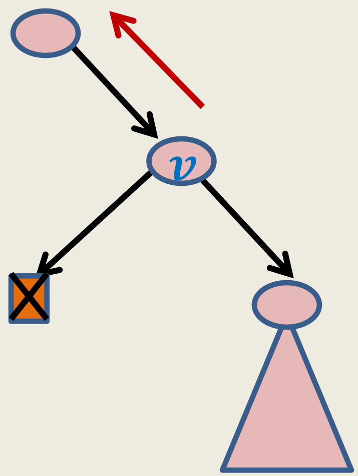
Let v be the **node** of **T** storing value x.



Case 2: **left**(v) == **NULL** , then **Predecessor(T,x)** is ?

Predecessor(T,x)

Let v be the **node** of **T** storing value x.

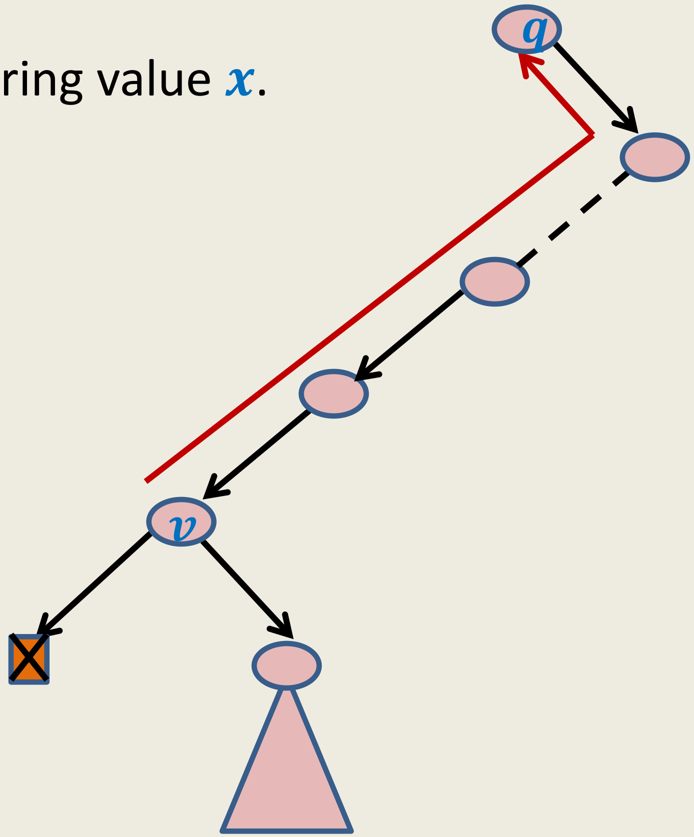


Case 2: **left**(v) == **NULL** , and v is **right child** of its **parent**

then **Predecessor(T,x)** is **parent**(v)

Predecessor(T,x)

Let v be the **node** of **T** storing value x.



Case 3: **left**(v) == **NULL** , and v is **left child** of its **parent**

then **Predecessor(T,x)** is ?

Predecessor(T,x)

```
Predecessor(T,x)
{   Let v be the node of T storing value x.
    If (left(v) <> NULL)  then return  Max(left(v))
    else
        if (v = right (parent(v)) return  parent(v)
        else
        {
            while(v = left (parent(v))
                v ← parent(v);
            return parent(v);
        }
}
```

Predecessor(T,x)

```
Predecessor(T,x)
{   Let v be the node of T storing value x.
    If (left(v) <> NULL)  then return  Max(left(v))
    else
    {       while(v = left (parent(v))
                v ← parent(v);
            return parent(v);
    }
}
```

Homework 1: Modify the code so that it runs even when x is minimum element.

Homework 2: Modify the code so that it runs even when $x \notin$ T .

Successor(T, x)

The **smallest** element in **T** which is **bigger than x**

Red Black tree
(Height Balanced BST)

Operations you already know

1. **Search(T,x)**
2. **Insert(T,x)**
3. **Delete(T,x)**
4. **Min(T)**
5. **Max(T)**
6. **Predecessor(T,x)**
7. **Successor(T,x)**

A NOTATION

T < T' :

every element of **T** is <u>smaller</u> than every element of **T'**.

New operations

8. SpecialUnion(T, T'):

Given **T** and **T'** such that **T < T'**,

compute **T*=TUT'**.

NOTE: **T** and **T'** don't exist after the union.

9. Split(T, x):

Split **T** into **T'** and **T''** such that **T'** < **x** < **T''**.

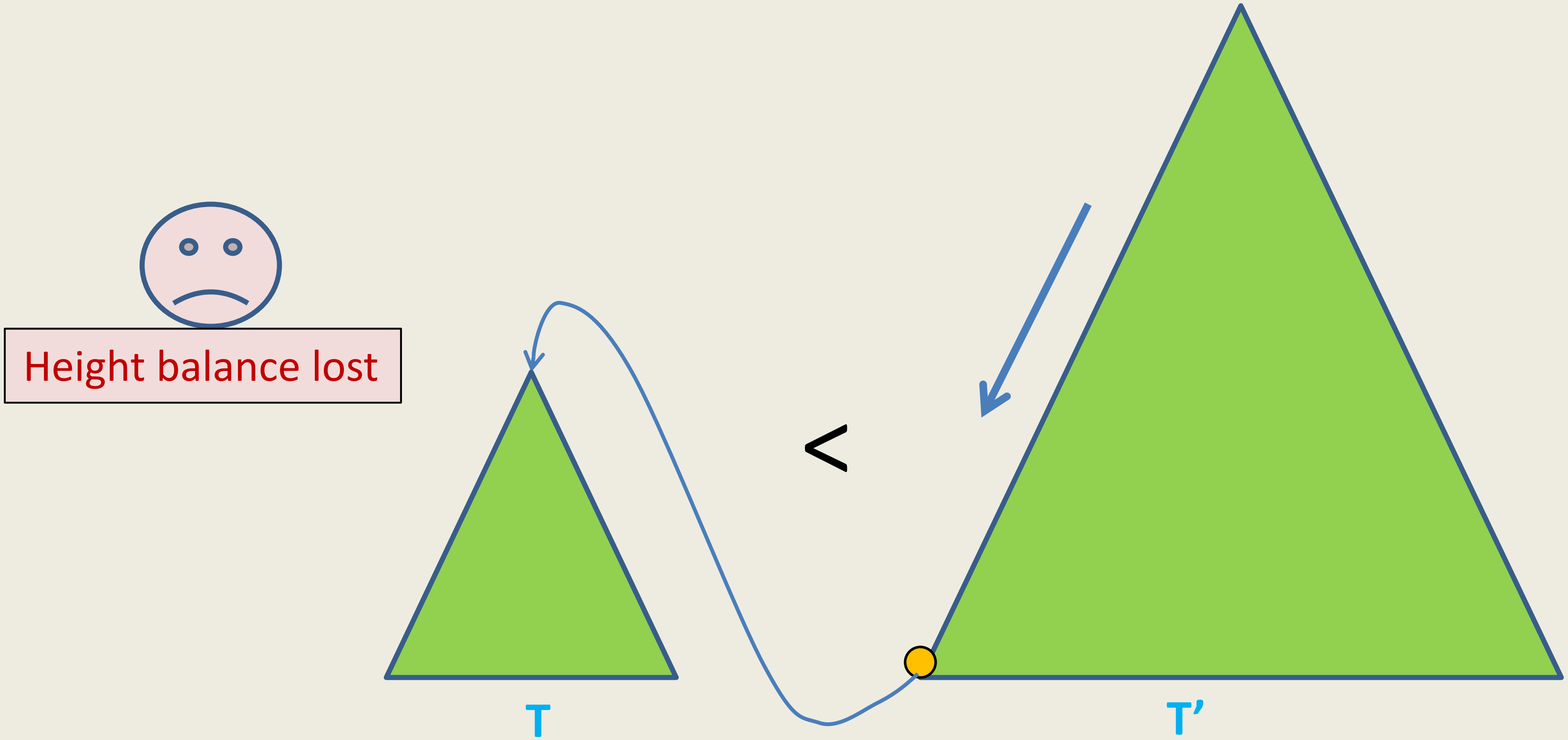
Every operation in **O**(log **n**) time.

SpecialUnion(T,T')

Remember:

every element of T is <u>smaller</u> than every element of T'

A trivial algorithm that does not work



Time complexity: O(log n)

Towards an $O(\log n)$ time for SpecialUnion(T,T') …

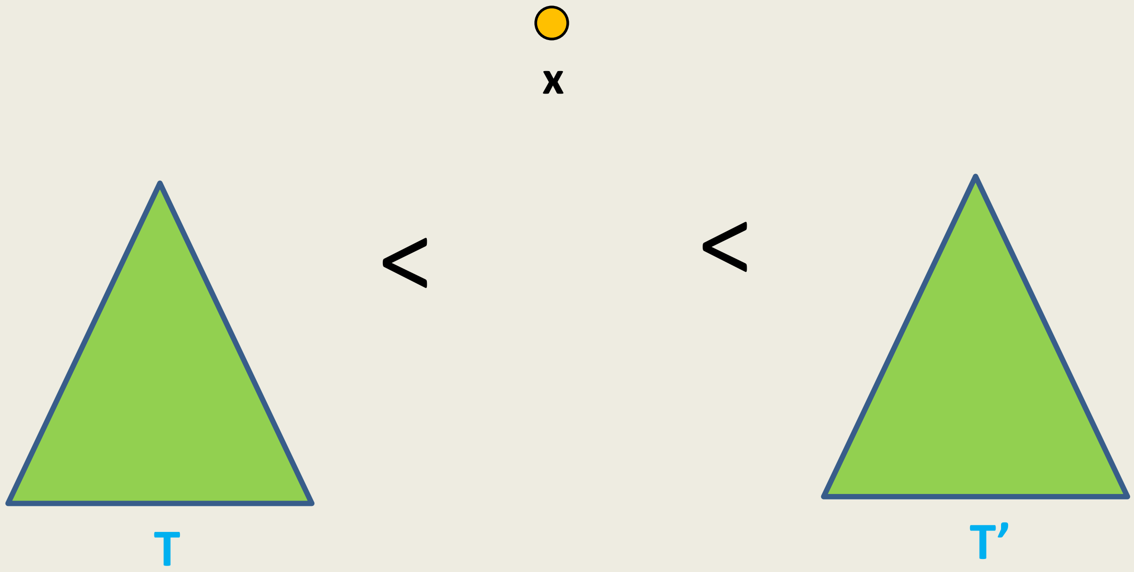
- **Simplifying the problem**

  Can we solve some special cases easily ?

- **Solving the simpler version efficiently**

- **Extending the solution to generic version**

Simplifying the problem

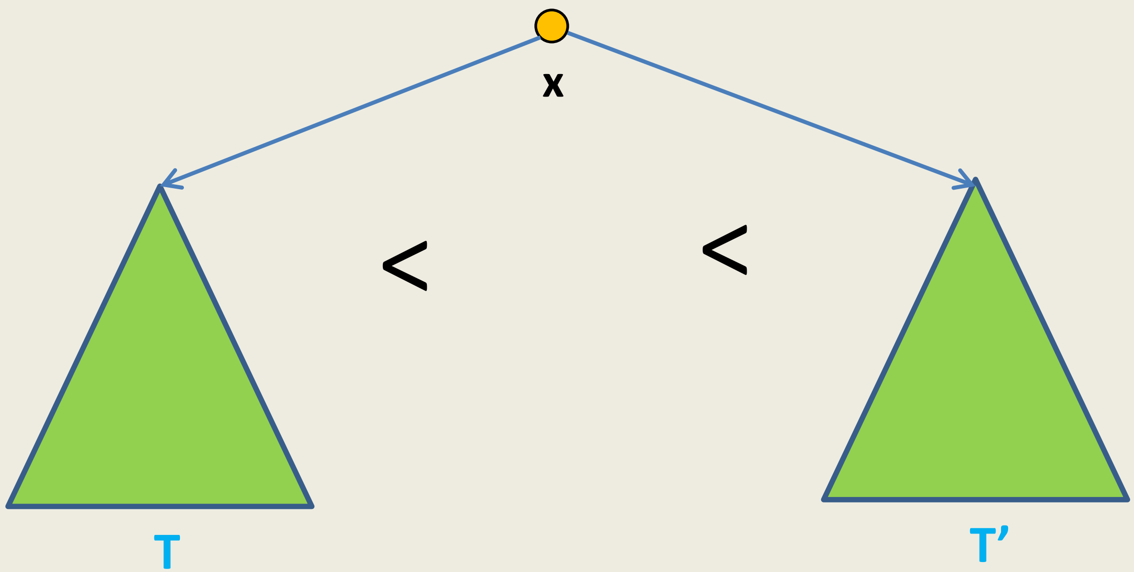


Simplified problem:

Given two trees T, T' of <u>same</u> **black height** and a key **x**, such that T<**x**<T', transform them into a tree T*=TU{**x**}UT'

Solving the simplified problem

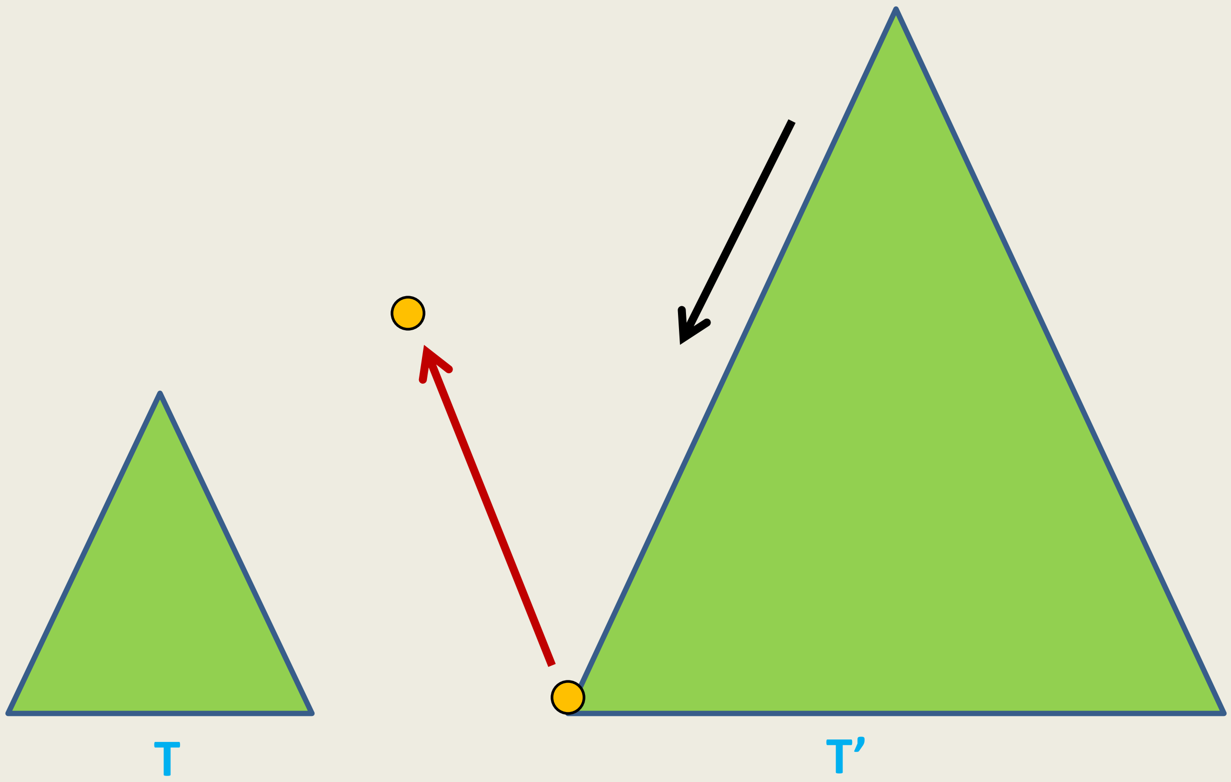
O (1) time



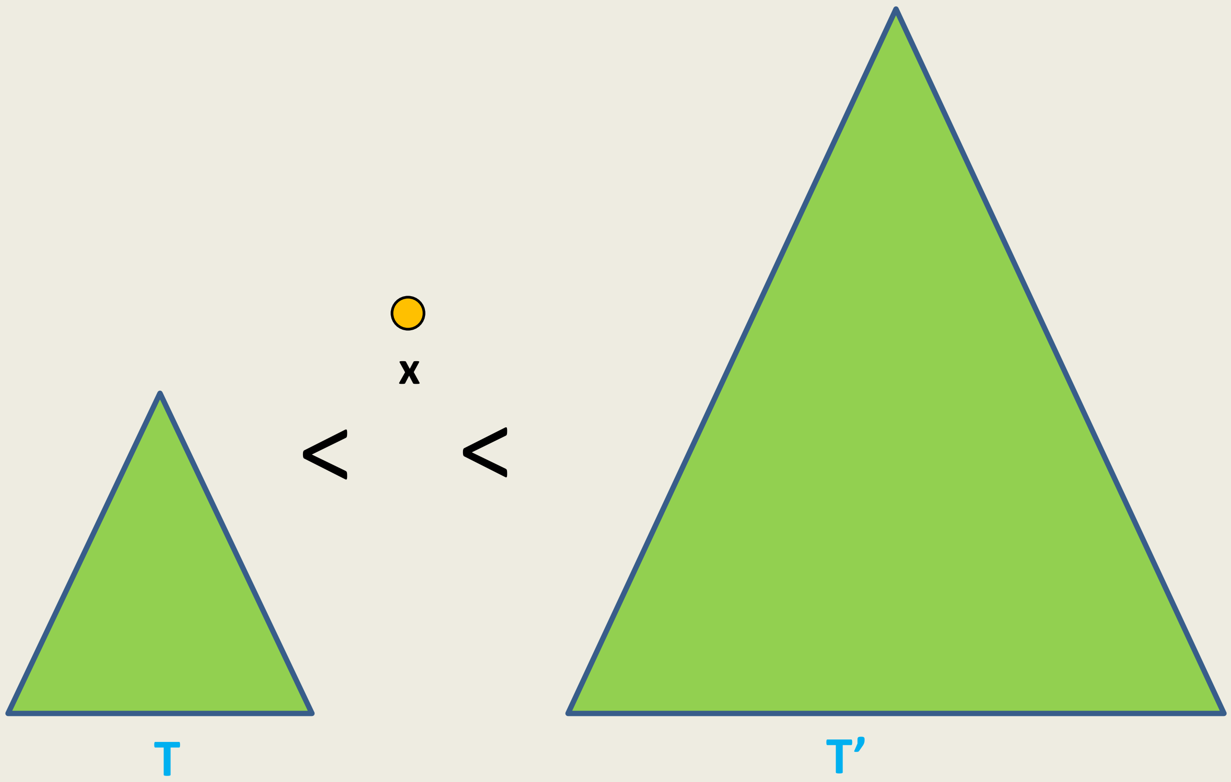
Simplified problem:

Given two trees **T**, T' of <u>same</u> **black height** and a key **x**, such that **T**<**x**<**T'**, transform them into a tree **T***=TU{**x**}U**T'**

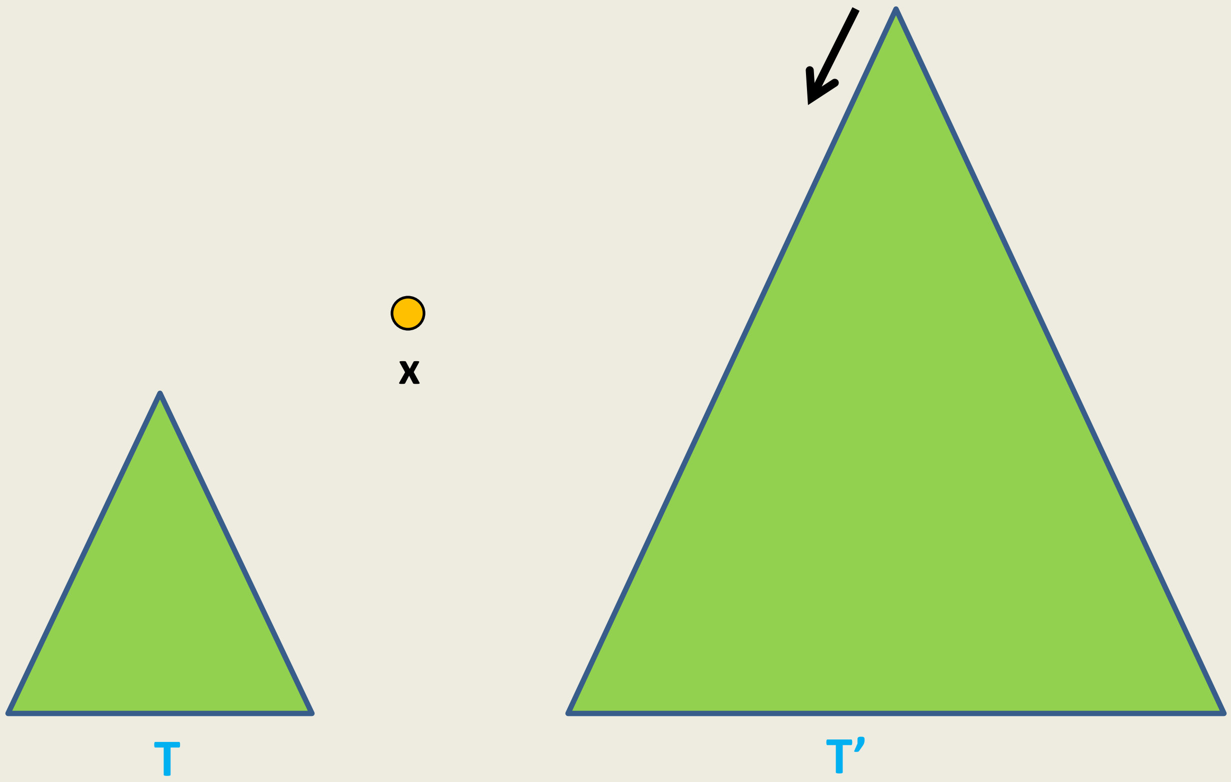
Extending the algorithm to the generic problem



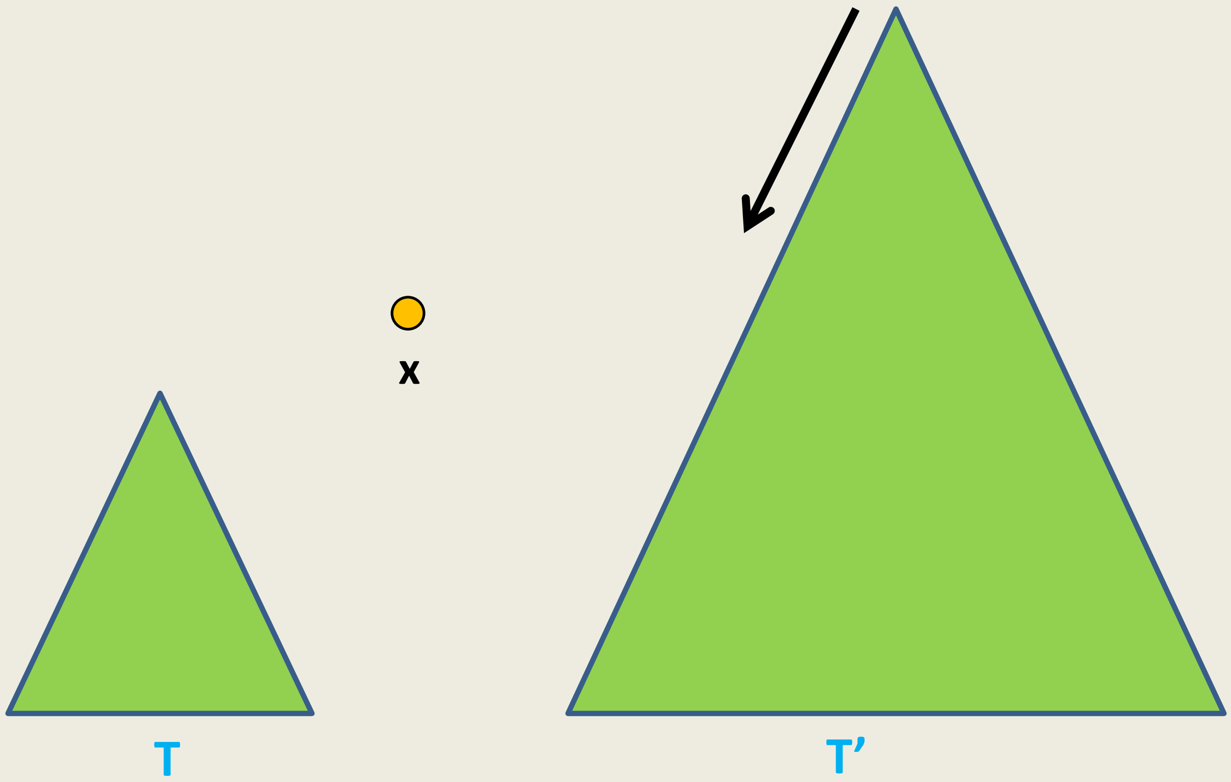
Extending the algorithm to the generic problem



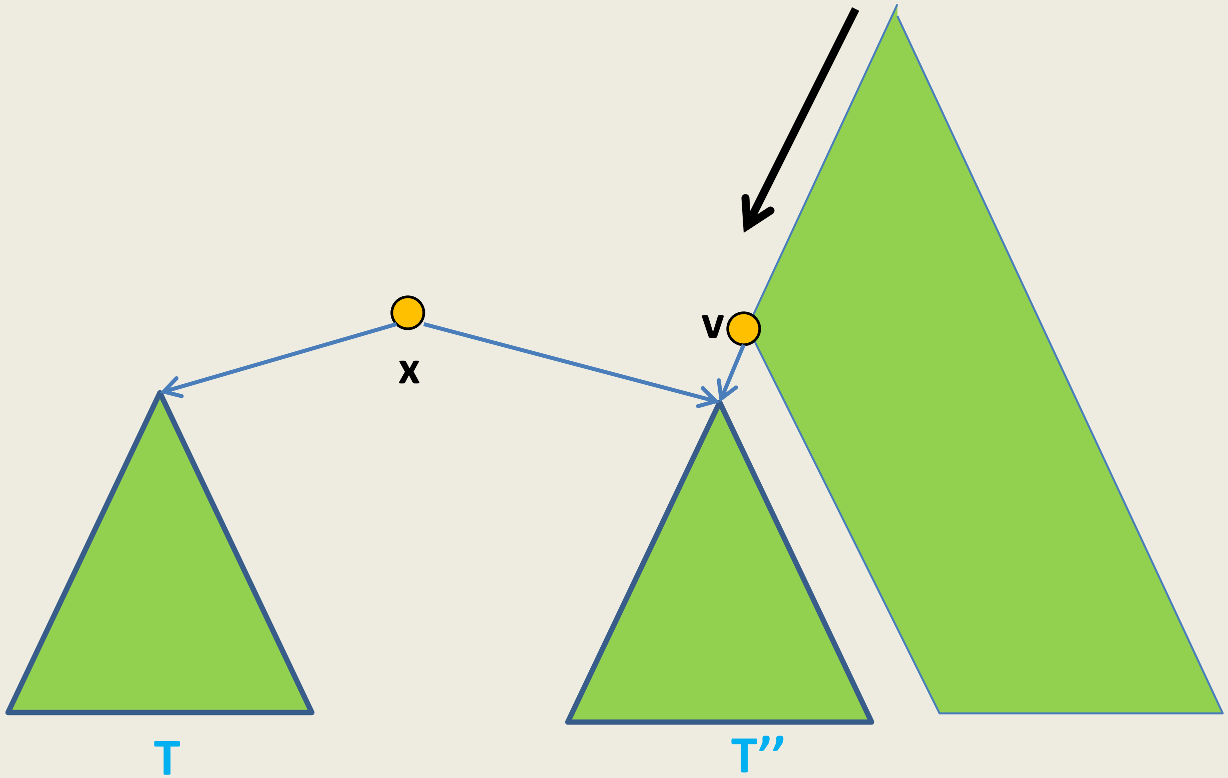
Extending the algorithm to the generic problem



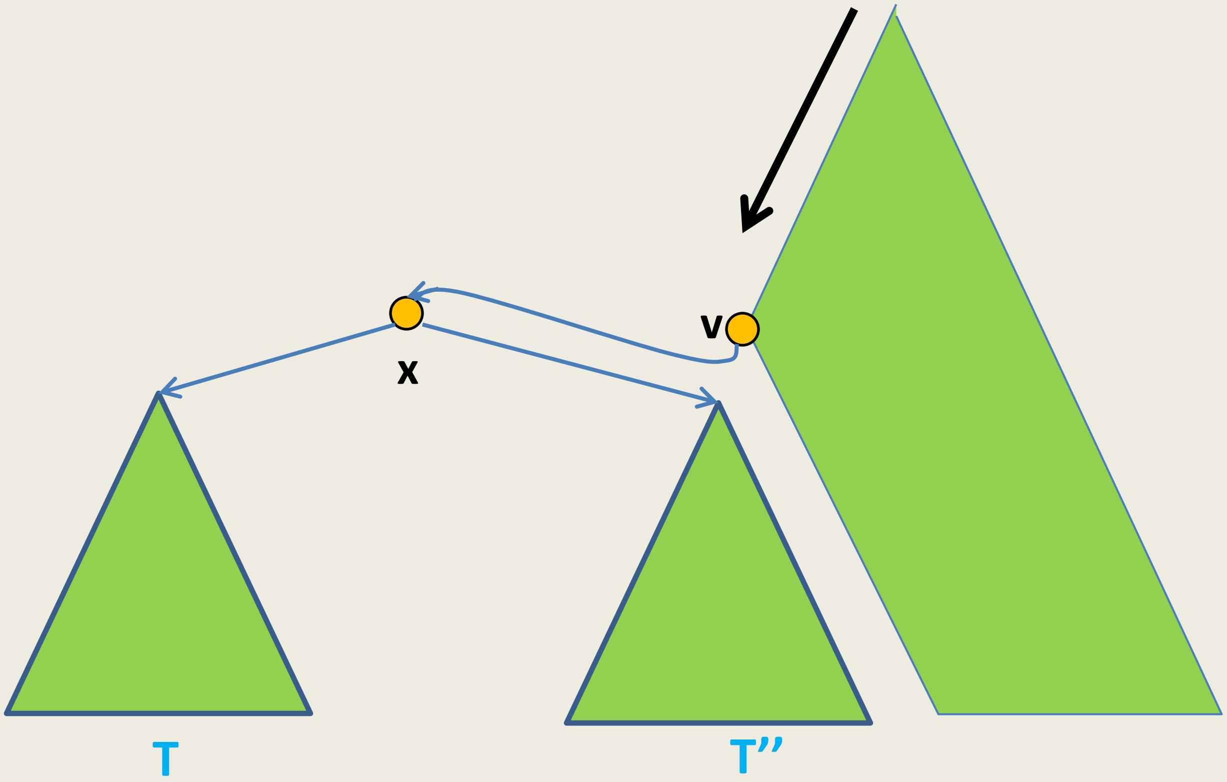
Extending the algorithm to the generic problem



Extending the algorithm to the generic problem



Extending the algorithm to the generic problem



Extending the algorithm to the generic problem

Algorithm for **SpecialUnion**(**T**,**T'**):

Let **black height** of **T** ≤ **black height** of **T'**

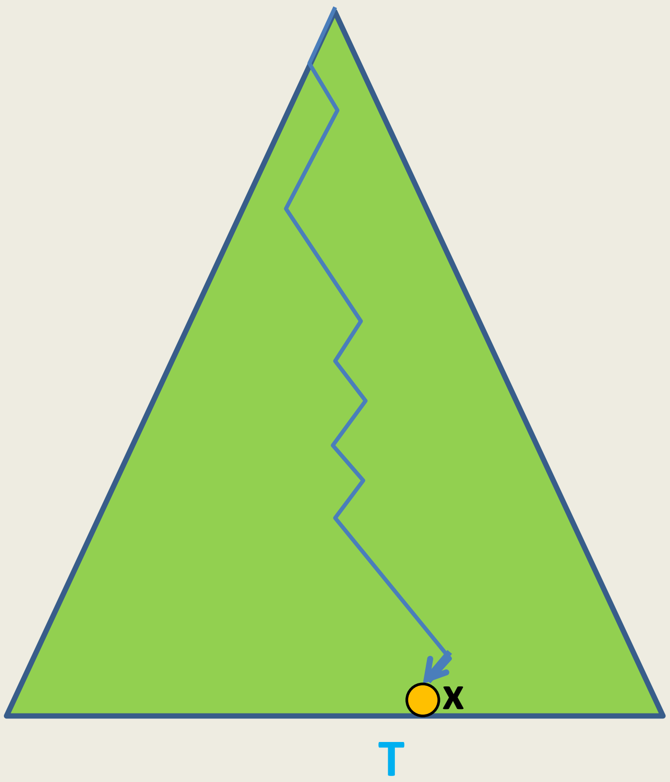
1. Let **x** be the node storing smallest element of **T'**.
2. **Delete** the node **x** from **T'.**
3. Keep following left pointer of **T'** until we reach a node **v** such that
    1. **left**(**v**) is black
    2. The subtree **T''** rooted at **Left**(**v**) has black height same as that of **T**
4. **left(x)← T;**
5. **right(x) ← T'';**
6. **Color(x)← red;**
7. **left(v)← x;**
8. **parent(x)← v;**
9. If **color(v)** is **red,** remove the color imbalance

    (like in the usual procedure of insertion in a **red-black** tree)

Total time : O(log n)

Split(T,x)

Achieving **O**(log **n**) time for **Split**(**T**,x)

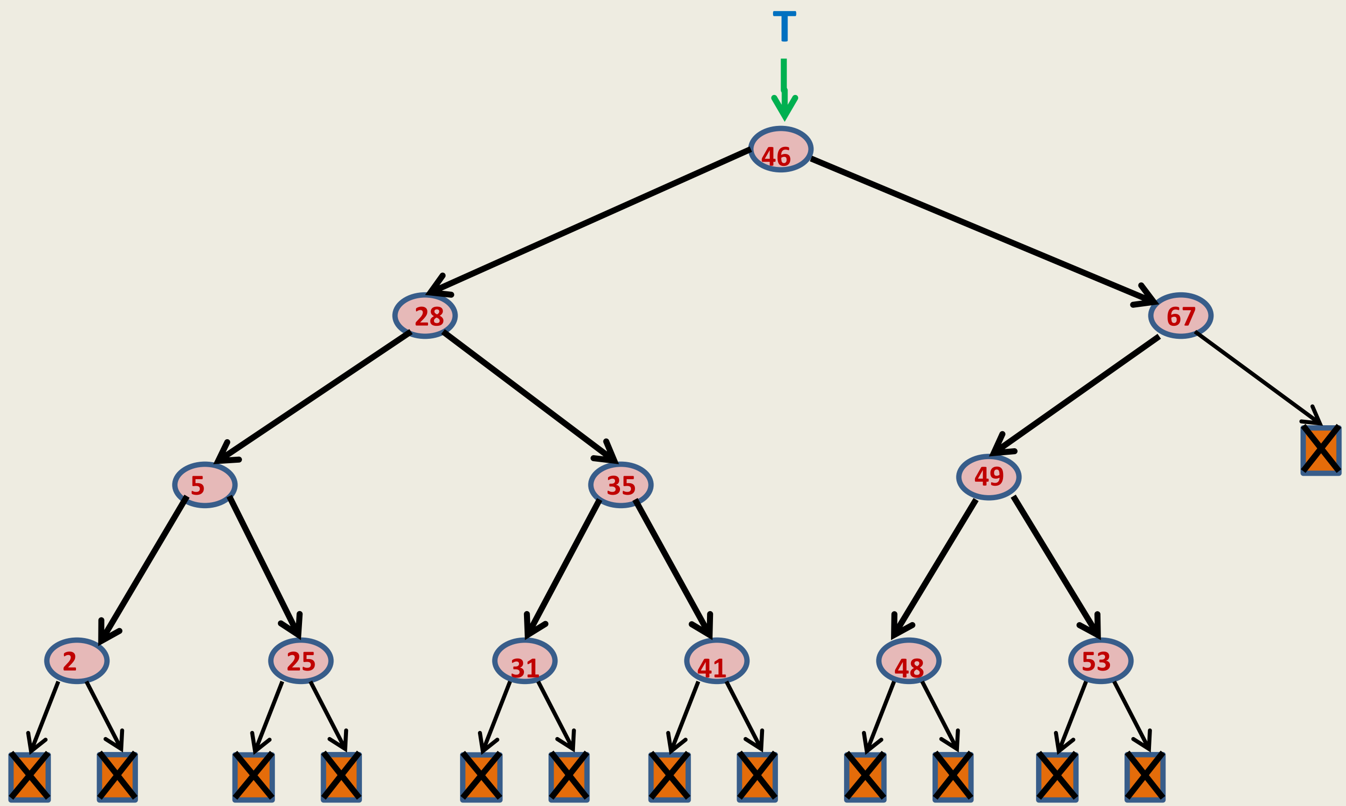


- Take a scissor and cut **T** into trees starting from **x**
- Make use of **SpecialUnion** algorithm.

Find-rank(T,x)

Return the **count** of no. of elements in **T** **smaller than** **x**

A trivial algorithm for Find-rank(T,x)



Repeatedly find predecessors of **x**

Time complexity: **O** (k log n)

(where k is rank of **x** in T)

O(log **n**) time is possible

Wait for one more month!!