Data Structures and Algorithms

(CS210/ESO207/ESO211)

Lecture 21

Graphs

- Definitions, notations, and terminologies
- Data structures for graphs
- A few algorithmic problems in graphs

Why Graphs ??

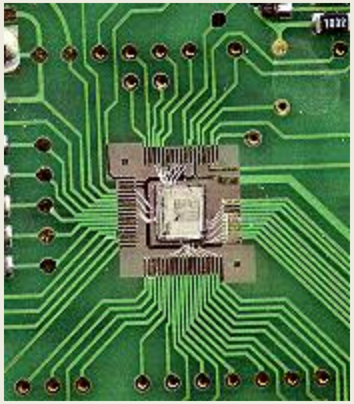
Finding shortest route between cities

Given a network of **roads** connecting various cities,

compute the shortest route between any two **cities**.

Just imagine how you would solve/approach this problem.

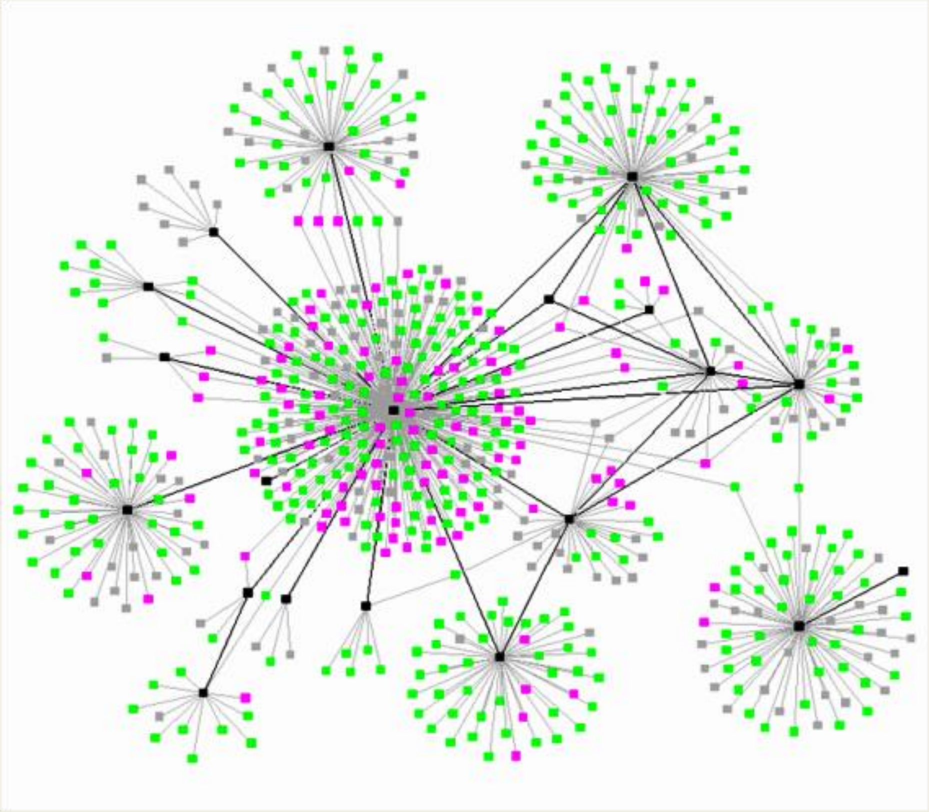
Embedding an integrated circuit on mother board



How to embed **ports** of various ICs on a plane and make **connections** among them so that

- No two connections intersect each other
- The total length of all the connections is minimal

A social network or world wide web (WWW)



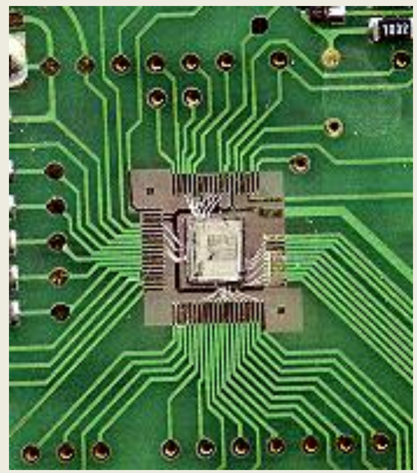
Can we make some useful observations about such networks ?

diameter

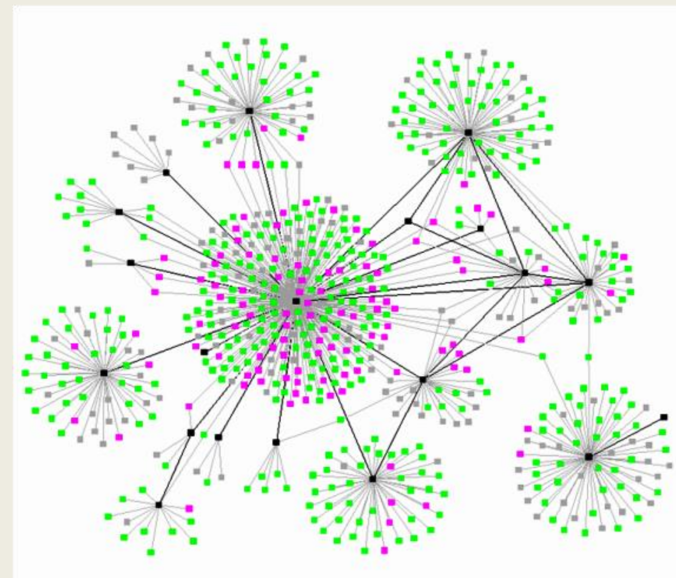
degree distribution

How will you model these problems ?

I

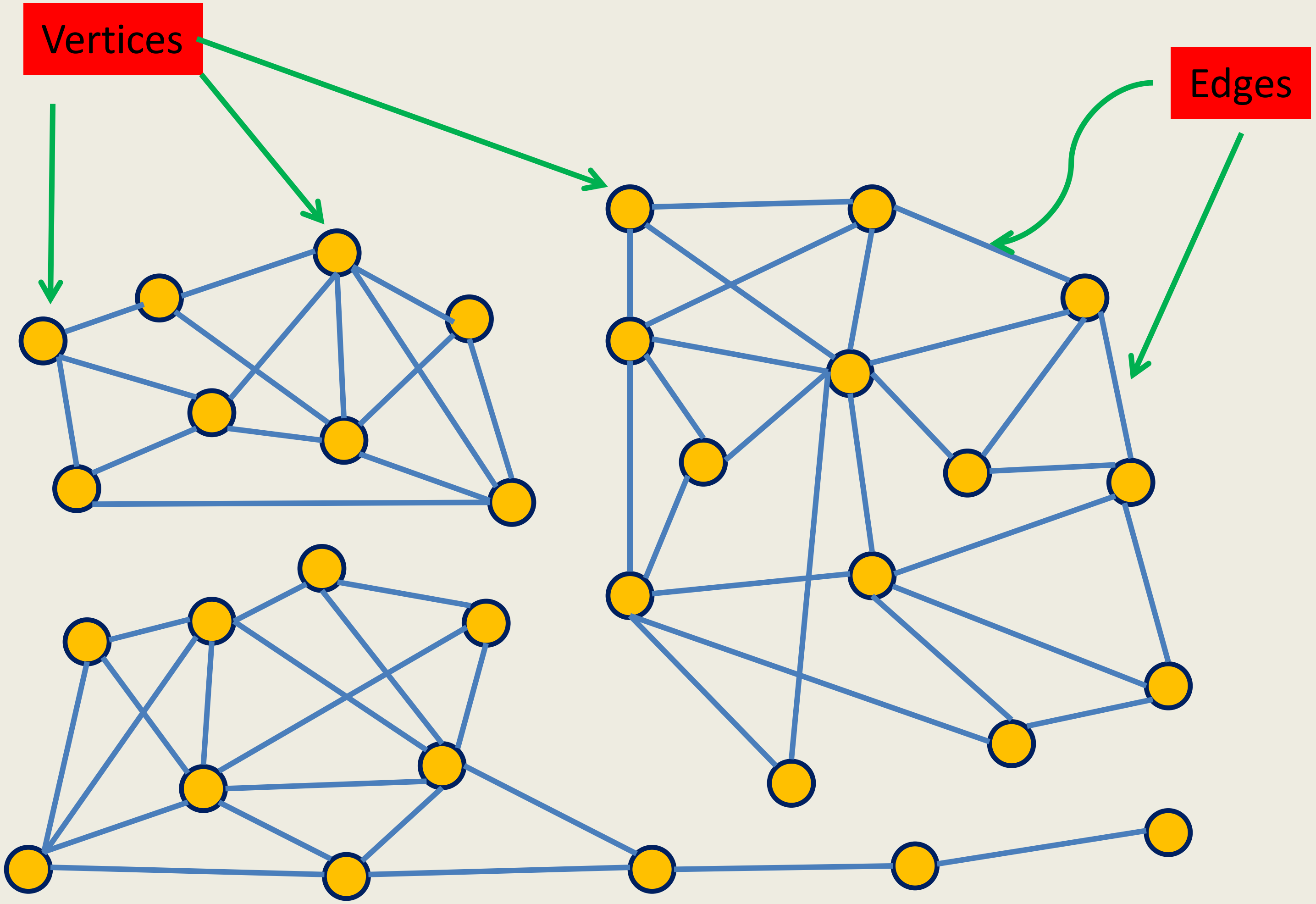


II



III

Graph



Graph

Definitions, notations, and terminologies

Graph

A graph G is defined by two sets

- V : set of vertices
- E : set of edges

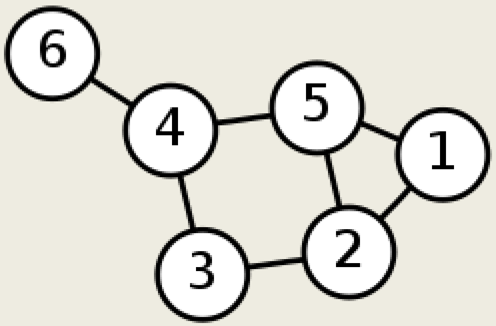
Question: any relation between E and V ? $E \subseteq (V \times V)$

Notation:

- A graph G consisting of vertices V and edges E is denoted by (V,E)

Types of graphs

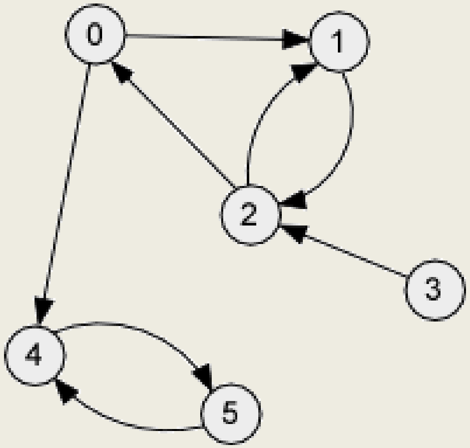
Undirected Graph



V= {1,2,3,4,5,6}

E= {(1,2), (1,5),
(2,5), (2,3),
(3,4),
(4,5), (4,6)}

Directed Graph



V= {0,1,2,3,4,5}

E= { (0,1), (0,4),
(1,2),
(2,0), (2,1),
(3,2),
(4,5),
(5,4) }

Notations

Notations:

- $n = |V|$
- $m = |E|$

Note: For directed graphs, $m \le$ $n(n-1)$

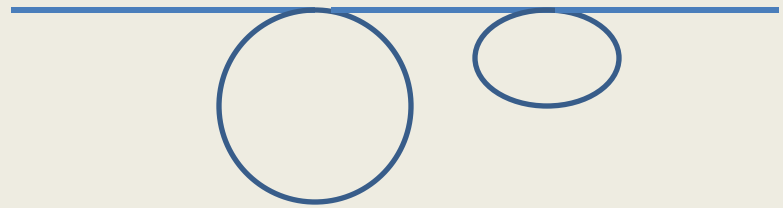
For undirected graphs, $m \le$ $n(n-1)/2$

Walks, paths, and cycles

Walk:

A sequence <v_0, v_1, …, v_k> of vertices is said to be a **walk** from x to y if

- $x = v_0$
- $y = v_k$
- For each $i<k$, $(v_i, v_{i+1}) \in E$

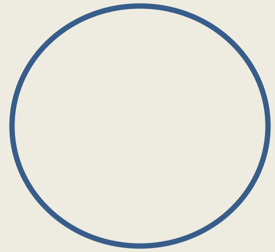


Path:

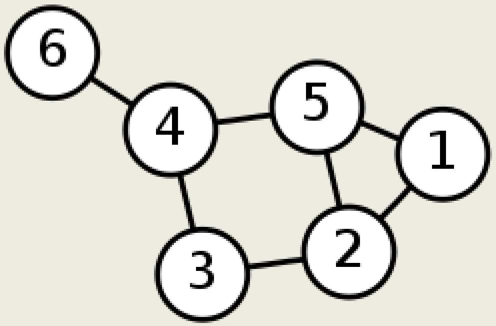
A walk <v_0, v_1, …, v_k> on which no vertex appears twice.

Cycle:

A walk <v_0, v_1, …, v_k> where no **intermediate** vertex gets repeated and $v_0 = v_k$

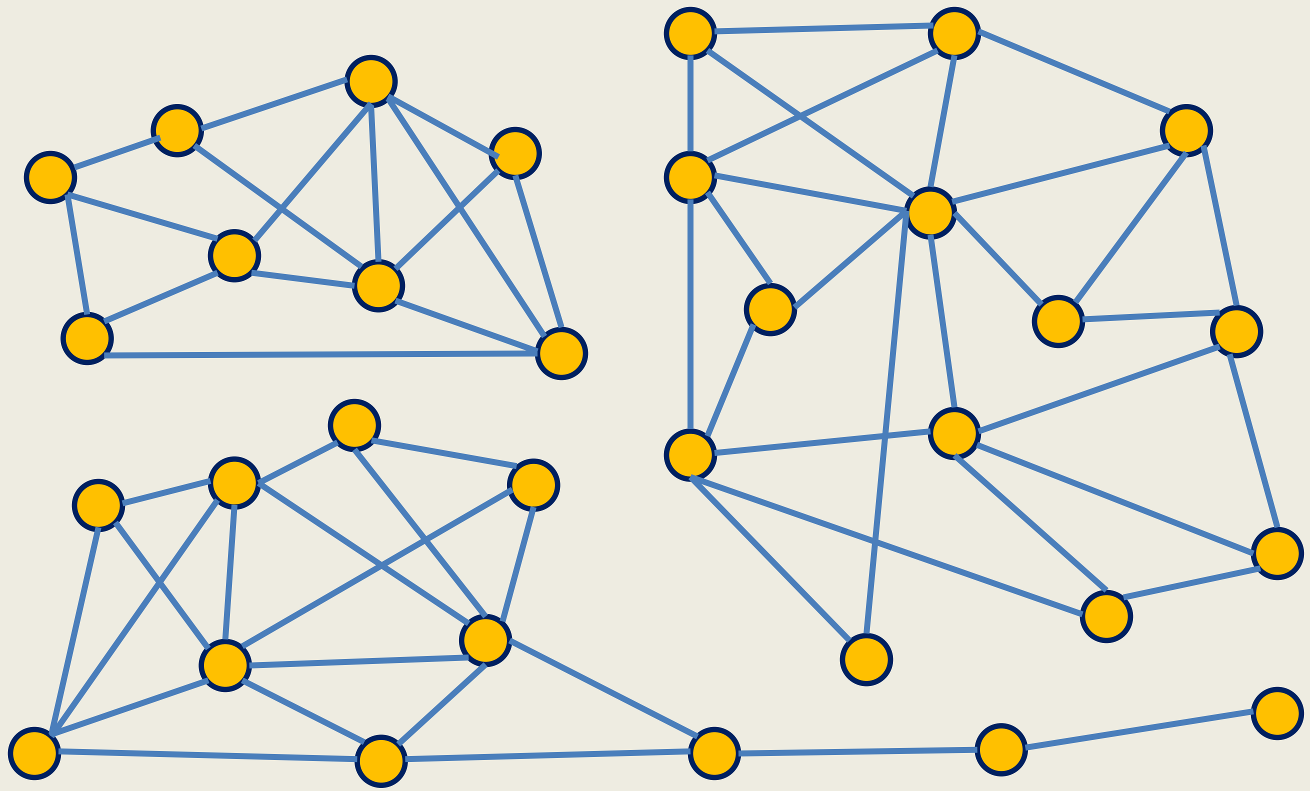


Terminologies



- **<1,5,4>** is a **walk** from **1** to **4.**
- <**1,3,2,5**> is **not** a **walk.**
- <**1,2,5,2,3,4,5,4,6**> is a **walk** from **1** to **6.**
- <**1,2,5,4,6**> is a **path** from **1** to **6.**
- <**2,3,4,5,2**> is a **cycle.**

Terminologies



two vertices are said to be ***connected*** if there is a **path** between them.

Connected component:

A **maximal** subset of connected vertices

You can not add any more vertex to the subset and still keeping it connected.

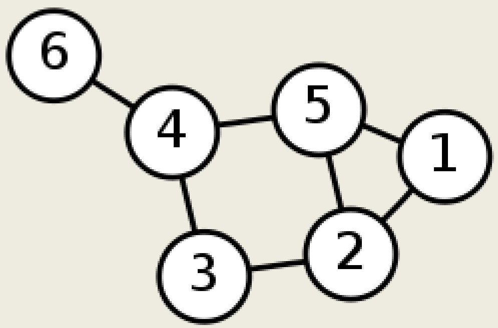
Data Structures for Graphs

Vertices are always numbered

$\mathbf{1, \dots, n}$ **Or** $\mathbf{0, \dots, n-1}$

Link based data structure for graph

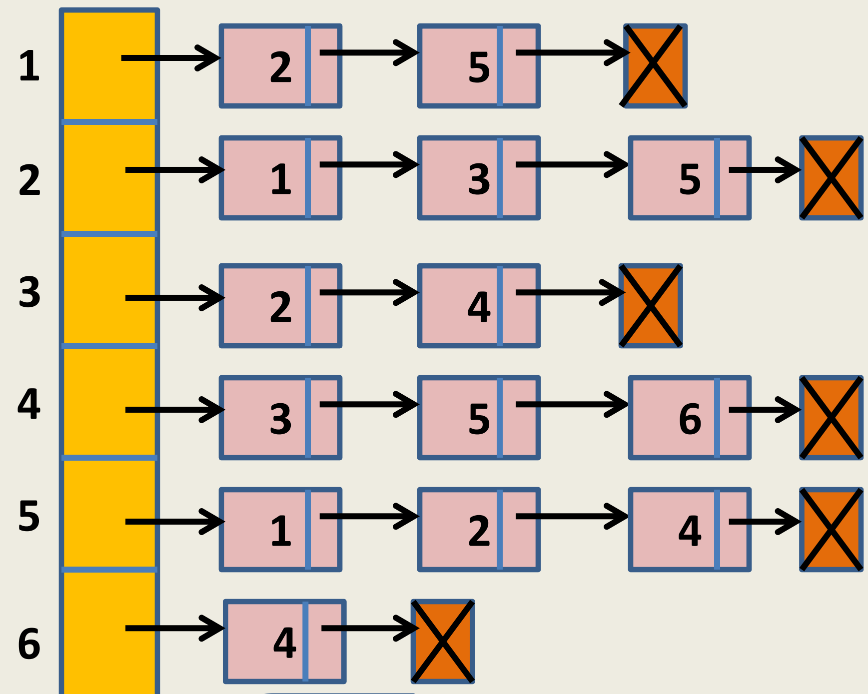
Undirected Graph



V= {1,2,3,4,5,6}

E= {(1,2), (1,5),
(2,5), (2,3),
(3,4),
(4,5), (4,6)}

Adjacency Lists



Size = O($n + m$)

Link based data structure for graph

Advantage of Adjacency Lists :

- Space efficient
- All the neighbors of a vertex can be computed in optimal time.

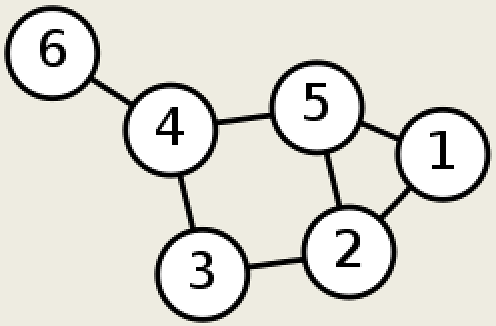
Disadvantage of Adjacency Lists :

- How to determine if there is an edge from ***x*** to ***y*** ?

(**O**(n) time in the worst case).

Array based data structure for graph

Undirected Graph



V= {1,2,3,4,5,6}

E= {(1,2), (1,5),
(2,5), (2,3),
(3,4),
(4,5), (4,6)}

Adjacency Matrix

	1	2	3	4	5	6
1	0	1	0	0	1	0
2	1	0	1	0	1	0
3	0	1	0	1	0	0
4	0	0	1	0	1	1
5	1	1	0	1	0	0
6	0	0	0	1	0	0

Size = O(n^2)

Array based data structure for graph

Advantage of Adjacency Matrix :

- It takes **O**(1) time to determine whether there is an edge from ***x*** to ***y*** for any two vertices ***x*** and ***y***.

Disadvantage of Adjacency Matrix :

- It takes **O**(n) time to compute/enumerate all neighbors of a given vertex ***x***.
- It takes **O**(n^2) space.

Which data structure is commonly used for storing graphs ?

Adjacency lists

Reasons:

- Graphs in real life are sparse ($m \ll n^2$).
- Most algorithms require enumerating and processing neighbors of each vertex.

➔Adjacency matrix will enforce **O**(n^2) bound on time complexity for the algorithm.

But, there are a few **exceptions**

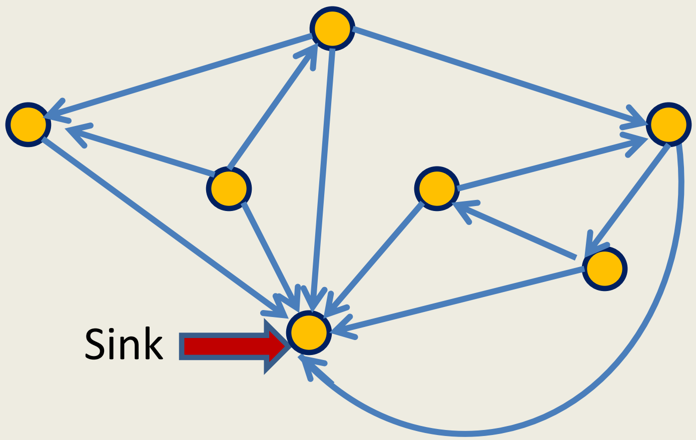
A sample of Graph algorithmic Problems

- Are two vertices x and y connected ?
- Find all connected components in a graph.
- Is there is a cycle in a graph ?
- Compute a path of shortest length between two vertices ?
- Is there is a cycle <u>passing through all vertices</u> ?

An interesting problem
(Finding a **sink**)

A vertex ***x*** in a given directed graph is said to be a **sink** if

- There is no edge emanating from (leaving) ***x***
- Every other vertex has an edge into ***x***.



Given a directed graph ***G***=(***V***,***E***) in an **adjacency matrix** representation, design an **O**(n) time algorithm to determine if there is any **sink** in ***G***.