Data Structures and Algorithms

(CS210A)

Lecture 37

- A new algorithm design paradigm: Greedy strategy

part IV

Problems solved till now

1. **Job Scheduling Problem**

2. **Mobile Tower Problem**

3. **MST**

Did you notice anything commons in their solutions ?

Ponder over this question before moving ahead

Problem 1

Job scheduling Problem

INPUT:

- A set J of n jobs $\{j_1, j_2, \ldots, j_n\}$
- job j_i is specified by two real numbers

  s(i): start time of job j_i

  f(i): finish time of job j_i
- A single server

We could not say anything about the **complete solution** of this problem.

Constraints:

- Server can execute at most one job at any moment of time and a job.
- **Job** j_i, if scheduled, has to be scheduled during[s(i), f(i)] only.

Aim:

To select the **largest** subset of **non-overlapping** jobs which can be executed by the server.

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To select the **largest** subset of **non-overlapping** jobs which can be executed by the server.

All that we could do was to make a **local** observation

Let $x \in J$ be the job with earliest finish time.

Lemma1 : There exists <u>an</u> optimal solution for J in which x is present.

Let $J' = J \backslash$ **Overlap**(x)

Lemma 1 gives very small information about the optimal solution ☹
How to use it to compute this solution ?

J (original instance)

Greedy step

J' (**<u>smaller</u>** instance)

Opt(J)

Lemma1

Opt(J')

Opt(J)= Opt(J') + 1 -- (i)

Equation **(i)** hints at recursive solution of the problem ☺

Theorem: $\mathbf{Opt}(J) = \mathbf{Opt}(J') + 1$.

- Proof has two parts

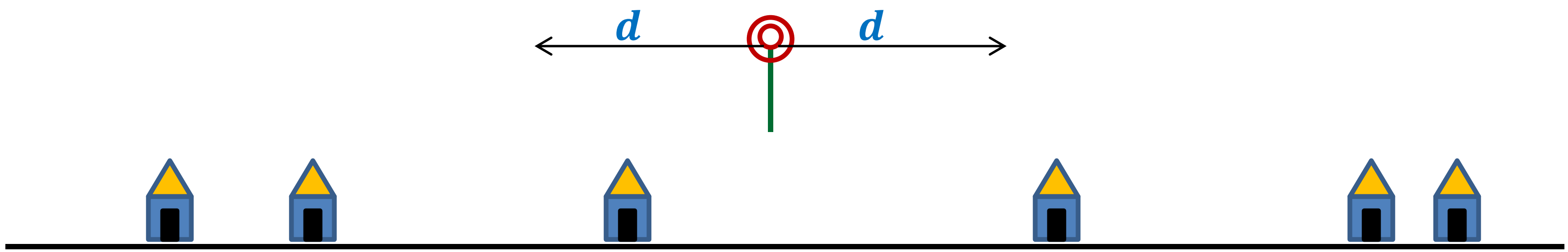
  $\mathbf{Opt}(J) \geq \mathbf{Opt}(J') + 1$

  $\mathbf{Opt}(J') \geq \mathbf{Opt}(J) - 1$

- Proof for each part is a proof by **construction**

Problem 2

Mobile Tower Problem



We could not say anything about the **complete solution** of this problem.

Problem statement:

There is a set H of n houses located along a road.

We want to place mobile towers such that

- Each house is **covered** by at least one mobile tower.
- The number of mobile towers used is **least** possible.

All that we could do was to make a local observation

Lemma 2: There is an optimal solution for the problem in which the leftmost tower is placed at distance d to the right of the first house.

Let x be the tower loc

Let H' = $H \backslash$ {all hous

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How to use it to compute this solution ?

H(original instance)

Opt(H)

Opt(H)= **Opt**(H') + **1**

Greedy step

Lemma 2

H'(**smaller** instance)

Opt(H')

Equation **(i)** hints at recursive solution of the problem ☺

What is a greedy strategy ?

A strategy that is

- Based on some **local** approach

- With the **objective to optimize** some function.

Note:

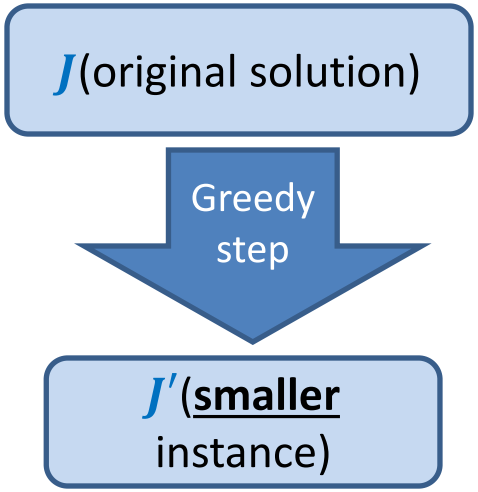
Recall that the divide and conquer strategy takes a **global approach**.

Design of a greedy algorithm

Let **A** be an instance of an optimization problem.

1. Make **a local observation** about the solution.

2. Use this observation to express optimal solution of **A** in terms of
   - Optimal solution of a smaller instance **A'**
   - Local step



3. This gives a recursive solution.
4. Transform it into iterative one.

MST

Input: an undirected graph G=(V,E) with **w**: $E \rightarrow \mathbb{R}$,

Aim: compute a **spanning tree** (V, E'), $E' \subseteq E$ such that $\sum_{e \in E'} \mathbf{w}(e)$ is **minimum.**

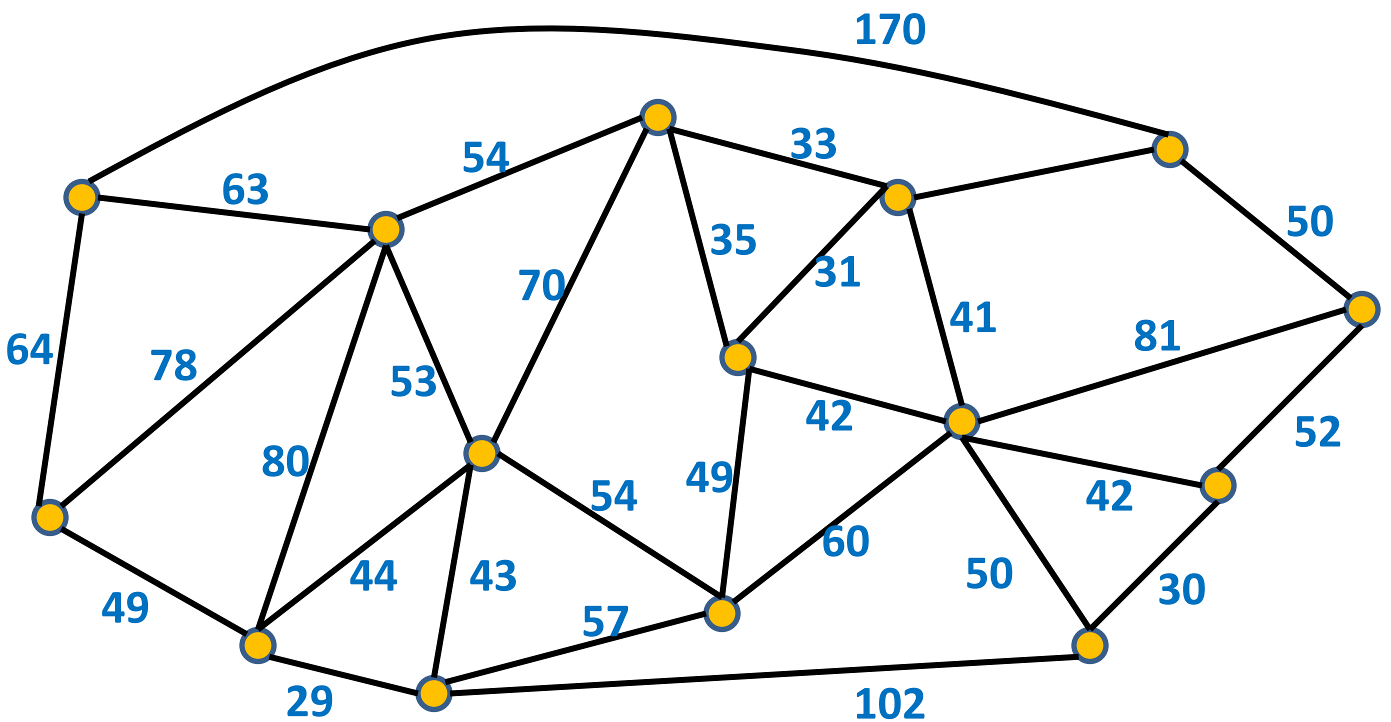
If you have understood a generic way to design a greedy algorithm, then try to solve the MST problem.

Lemma 2

If $e_0 \in E$ is the edge of **least weight** in G, then there is a **MST** T containing e_0.

How to use this **Lemma** to design an algorithm for **MST** ?

How to compute a MST ?



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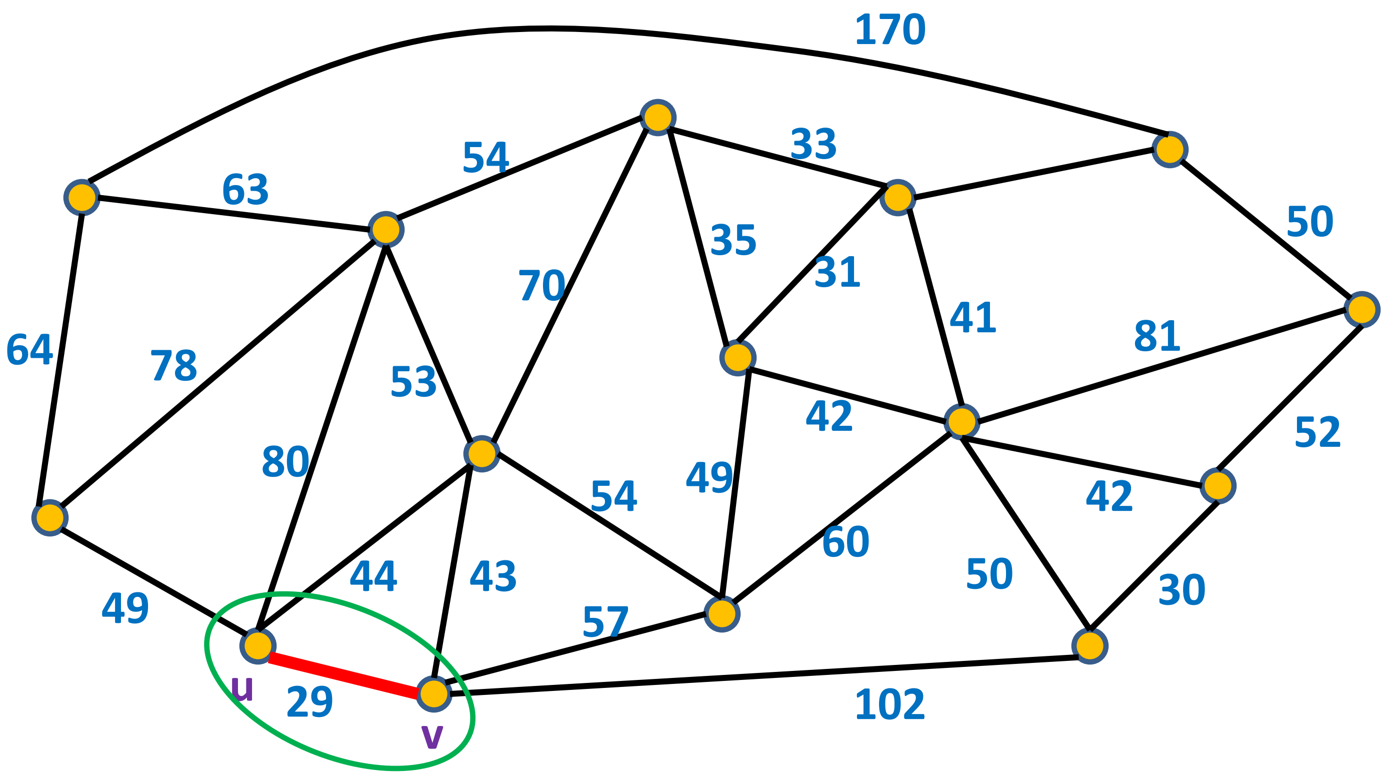

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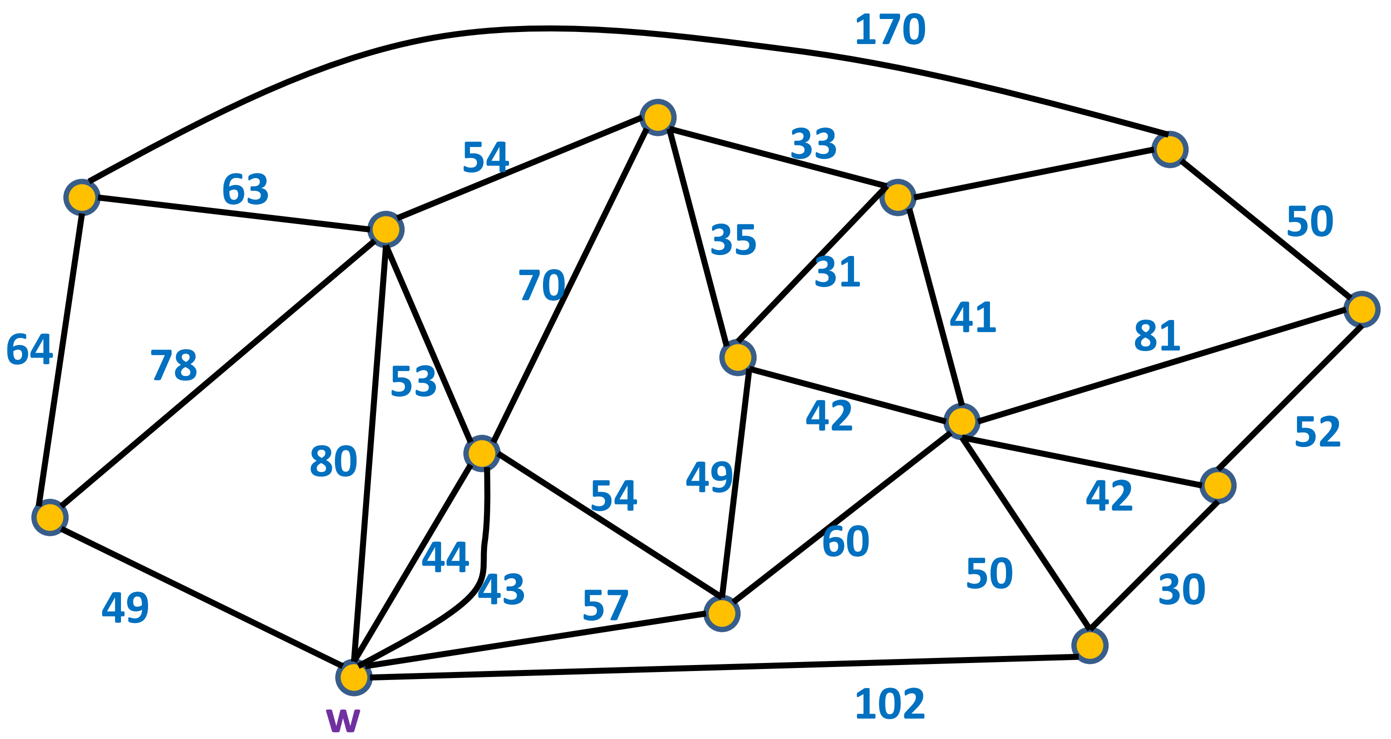


This is graph G'

How to compute a MST ?

Let (u,v) be the least weight edge in G=(V, E). Transform G into G' as follows.

- **Remove** vertices u and v and **add** a new vertex w
- For each edge (u,x)ϵ E, add edge (w,x) in G'.
- For each edge (v,x)ϵ E, add edge (w,x) in G'.
- In case of multiple edges between w and x, keep only the **lighter** weight edge.

Theorem1: **W_MST(G) = W_MST(G') + w(u,v)**

Proof: (by **construction)**

1. W_MST(G) ≤ W_MST(G') + w(u,v) ⟸ straightforward

2. W_MST(G') ≤ W_MST(G) - w(u,v) ⟸ Use **Lemma 3**

(Give all details of the proof as a homework)

Problem 4
Overlapping Intervals

The aim of this problem is to make you realize that

it is sometime very nontrivial to design a greedy algorithm. In particular, it is quite challenging to design the smaller instance.

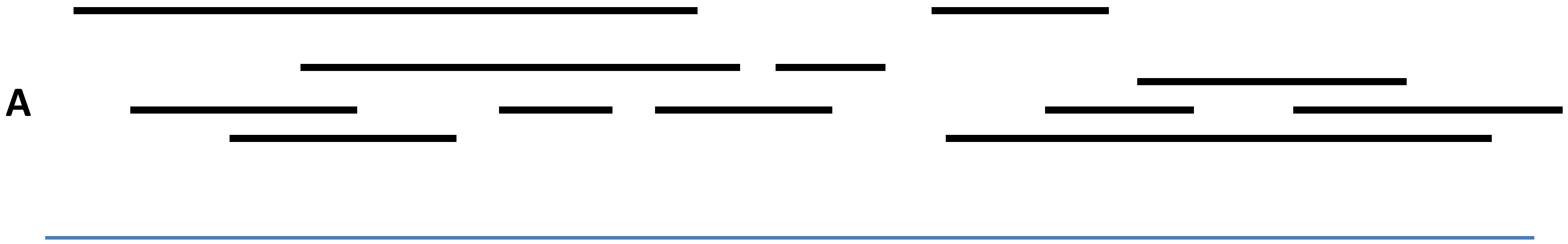
In the end semester exam of the course, no problem of this level of difficulty will be asked ☺

Problem 4
Overlapping Intervals

Overlapping Intervals

Problem statement:

Given a set **A** of n intervals, compute smallest set **B** of intervals so that for every interval **I** in **A\B**,

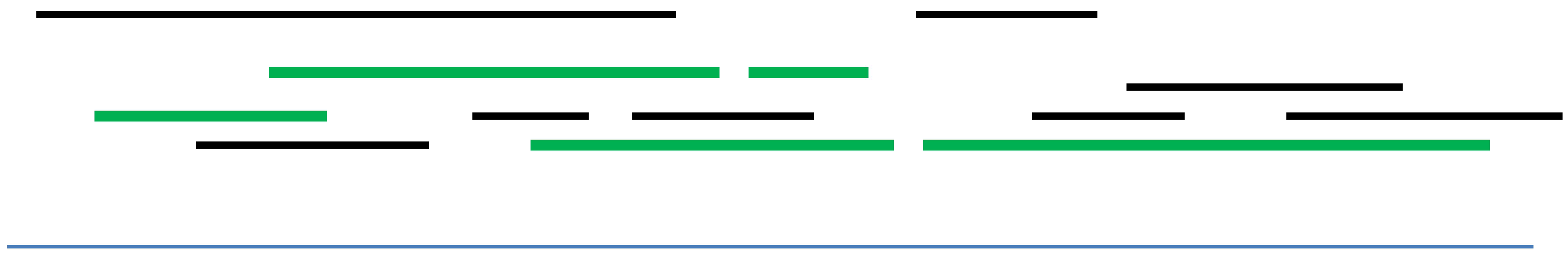


Overlapping Intervals

Problem statement:

Given a set **A** of n intervals, compute smallest set **B** of intervals so that for every interval **I** in **A\B**, there is some interval in **B** which overlaps/intersects with **I**.

A



another optimal solution ☺

Overlapping Intervals

Strategy 1

Interval with <u>**maximum length**</u> should be there in optimal solution

Intuition:

Selecting such an interval will **cover maximum** no. of other intervals

There is a counter example ☹

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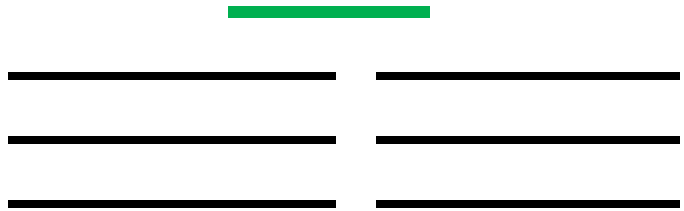
Overlapping Intervals

Strategy 2

Interval that **overlaps maximum no.** of intervals should be there in optimal solution



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Overlapping Intervals

Strategy 2

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Overlapping Intervals

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Interval that **overlaps maximum no.** of intervals should be there in optimal solution

Not an optimal solution ☹

Overlapping Intervals

Strategy 2

Interval that **overlaps maximum no.** of intervals should be there in optimal solution

An optimal solution has size 2.

Think for a while :

After failure of two strategies, how to proceed to design the algorithm.

Overlapping Intervals

Let **I*** be the interval with earliest finish time.

Let **I’** be the interval with **maximum** finish time overlapping **I***.

Lemma1: There is an optimal solution for set **A** that contains **I’.**

Proof:(sketch) :

If **I*** is overlapped by any other interval in the optimal solution, say **I^**,

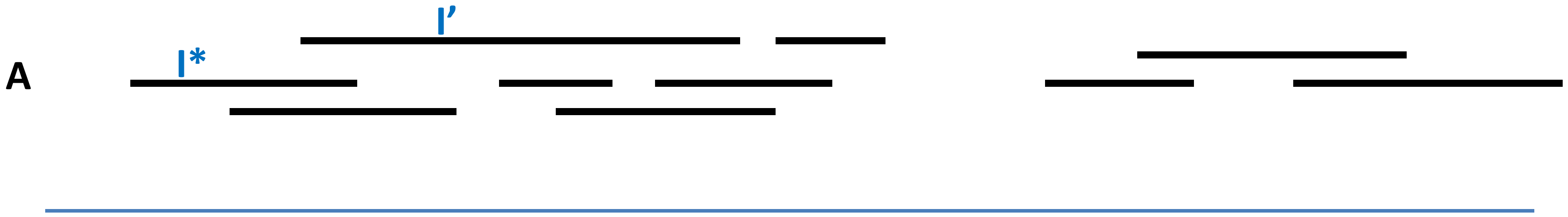
I’ will surely overlap all intervals that are overlapped by **I^.**

➔ Swapping **I^** by **I’** will still give an optimal solution.

Exploit the fact that **I*** has earliest finish time for this claim.

Overlapping Intervals

Question: How to obtain smaller instance **A'** using **Lemma 1** ?

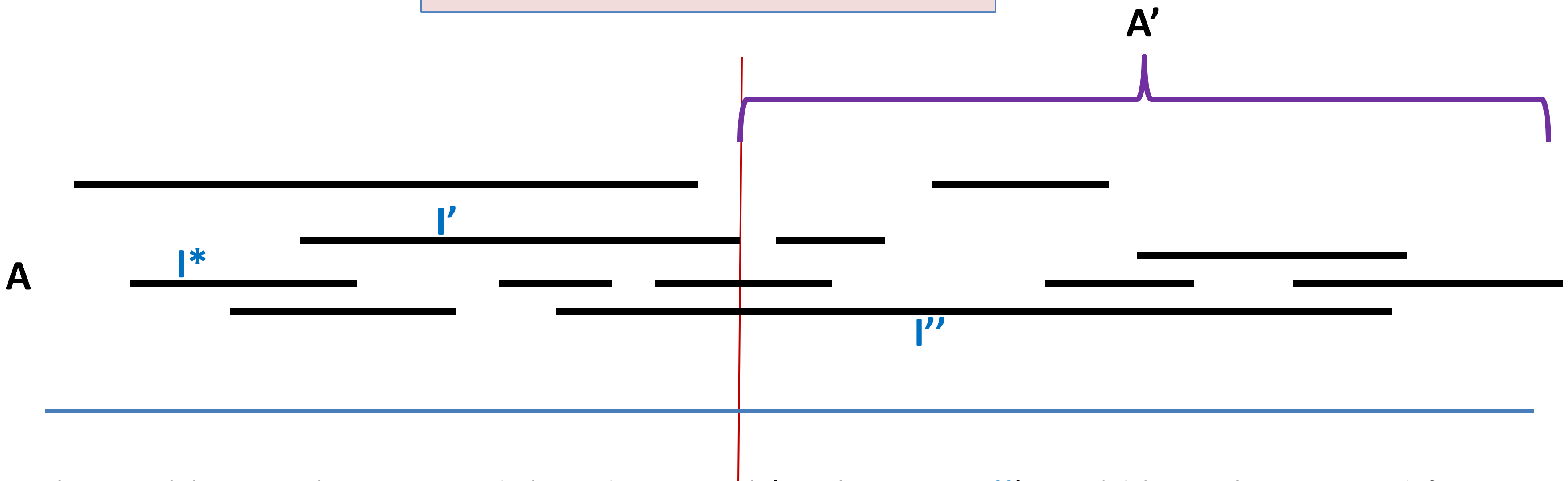


Overlapping Intervals

Question: How to obtain smaller instance **A'** using **Lemma 1** ?

Naive approach : remove from **A** all intervals which overlap with **I'**. This is **A'**.

There is a counter example ☹



The problem is that some deleted interval (in this case **I''**) could have been used for intersecting many intervals if it were not deleted. But deleting it from the instance disallows it to be selected in the solution.

Homework
for the summer break

- How will you form the smaller instance ?
- Design an algorithm for the problem.
- Give a neat, concise, and formal proof of correctness of the algorithm.

If you try <u>sincerely</u> and <u>don't give up</u> quickly,
you will surely be able to solve this problem.

We can discuss it during next semester in some doubt clearing session.
You have to bear with me for one more semester in CS345 ☺.